Problem assignment 3

Analysis on Manifolds

Joseph Bernstein November 26, 2003

1. Let X be a smooth compact manifold of dimension k.

(i) Show that there exists an immersion $\nu : X \to \mathbf{R}^{2k}$

(ii) Show that the exists a morphism $\nu : X \to \mathbf{R}^{2k-1}$ which is an immersion everywhere except finite number of points.

(ii) For every point $m \in M_r$ describe the tangent space $T_m(M_r)$

2. Consider a morphism of manifolds $\nu : X \to Y$ and a submanifold $Z \subset Y$. Let us fix a morphism $p : Y \to S$ where S is some manifold. Then over every point $s \in S$ we can consider fibers X_s, Y_s, Z_s and the morphism $\nu_s : X_s \to Y_s$.

Suppose we know that ν is transversal with respect to Z.

Show that for almost every point $s \in S$ the fibers are manifolds and the morphism ν_s is transversal with respect to a submanifold Z_s.

3.