

## Advanced Algebra 2

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In this course I will try to give an exposition of the Class Field Theory (CFT). This theory is one of the basic results obtained in number theory. It describes the structure of abelian Galois extensions of a number field  $K$  and their connection with the arithmetic of the field  $K$ .

### Short description of the course

1. I will start with detailed but fast review of Galois theory. This will include the elementary construction of Frobenius elements in Galois groups over  $\mathbb{Q}$ .
2. Galois theory of infinite algebraic extensions.
3. Cyclotomic extensions and their Galois groups.
4. Kronecker-Weber theorem and the structure of abelian Galois group of  $\mathbb{Q}$ .
5. Completions of  $\mathbb{Q}$  and number fields;  $p$ -adic fields.
6. Arithmetic of number fields and of  $p$ -adic fields.
7. Adeles and ideles
8. Class field theory - local and global.
9. Analytic methods in number theory - L-functions.

### Books

In my exposition I will mostly follow the book by J. Neukirch "Class field Theory".

I will also try to give a description of CFT for the case of geometric fields following approach by Serre.