Problem assignment 4

Algebraic Geometry and Commutative Algebra II

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1. Let X be a curve in $\mathbf{P}^2$ defined by a polynomial of degree d.

(i) Suppose X is non-singular. Can you compute its genus.

(ii) Suppose X is non-singular outside k points and at these points it has simplest nodal singularities.

Compute the arithmetic genus of X. Compute the geometric genus of X i.e. the genus of its smooth model.

Let V be a finite dimensional vector space over k and $\mathbf{V}$ the corresponding algebraic variety. We set $\mathbf{V}^* = \mathbf{V} \setminus 0$ and denote by j the imbedding $j : \mathbf{V}^* \to \mathbf{V}$.

2. Let F be an $\mathcal{O}$ module on $\mathbf{V}^*$.

(i) Show that $H^i(\mathbf{V}^*(F)) = \Gamma(\mathbf{V}, R^i j_*(F))$

(ii) Show that for $i > 0$ the action of the algebra $\mathcal{O}(\mathbf{V})$ on the cohomology space $H^i(\mathbf{V}^*(F))$ is locally nilpotent when restricted to the maximal ideal of the point 0.

3.. Let F be a coherent $\mathcal{O}$-module on $\mathbf{P}(V)$. Show that we can embed F into a coherent acyclic $\mathcal{O}$-module.

Show that we can find a resolution of F of length $\leq \dim V$ by coherent acyclic $\mathcal{O}$-modules.

4.. Let F be a coherent $\mathcal{O}$-module on $\mathbf{P}(V)$. Show that for large k the dimension $\dim \Gamma(\mathbf{P}(V), F(k))$ is a polynomial in k of degree equal to the dimension of support of F.