Differential Geometry

Joseph Bernstein Fall 2005

This is a basic course on differential geometry for toar rishon.

Preliminary material.

1. Reminder of basic notions of linear algebra over $\mathbf{R}$.
Linear space, basis, dimension, dual space.
Quadratic forms. Euclidean spaces.
2. Metric and topology on the Euclidean space $\mathbf{R}^n$.
Basic notions of metric spaces and topological spaces.
Local and global properties.

3. Differentiable functions on $\mathbf{R}^n$. Differentiable morphisms. Differentials.

4. Curves in Euclidean space $\mathbf{R}^n$. Curvature and torsion of a curve. Frenet formulas.

Basic notions of manifold theory.

5. Notion of a smooth manifold. Morphisms of manifolds.
Cut-off functions and partition of unity.
Local to global correspondence.

6. Inverse and implicit function theorems.
Immersions, submersions, morphisms of constant rank.
Submanifolds

7. Notion of tangent and cotangent spaces. Differentials.

8. Vector fields. Commutator.
Main theorem of ODE (the theory of ordinary differential equations).
Frobenius theorem.

Exterior differential calculus and integration theory.

9. Differential forms. De Rham differential. Functoriality.
Lie derivative, Weyl formulas.

10. Integration of 1-forms.
Orientation. Integration of differential forms. Change of variables.
Manifolds with boundary. Stokes' formula.
Classical formulations.
Applications of Stokes' formula.

11.General remarks on cohomologies. DeRham cohomology.
Poincare lemma and DeRham theorem.

Classical theory of surfaces in Euclidean 3-space.

12. First and second fundamental forms.

13. Principle curvatures. Gauss map and Gauss curvature. Intrinsic character of Gauss curvature.

Theory of vector bundles.

14. Vector bundles. Examples
Morphisms of vector bundles.
Subbundles, quotient bundles.
Splitting of an exact sequence of vector bundles.
Inverse image of vector bundles.

15. Vector bundles and principle bundles.
16. Reduction of the structure group.

17. Connections.
Algebraic description of connections.
Geometric description of connections.
Curvature of the connection

Basic notions of Riemannian geometry.

18. Notion of Riemannian manifold. Riemannian metric.
19. Levi-Civita connection.
20. Affine connections. Torsion and curvature. Geodesics of an affine connection. Exponential morphism.
21. Riemannian geodesics as extremal curves.

Classical results revisited.

Books.

In this course I will use the following books:

Spivak, Analysis on Manifolds.
Manfredo do Carmo, Differential Geometry of Curves and Surfaces
Chern, Chen, Lam, Lectures on Differential Geometry.