

## Algebraic theory of $D$ -modules

Joseph Bernstein

Nov 04, 2007

Algebraic theory of  $D$ -modules is the first part of a year long advanced course.

In this part I will study the properties of modules over the Weyl algebra  $D = D_n$  (which is just the algebra of linear differential operator in  $n$  variables).

This part will be rather elementary.

The main new and highly non-trivial notion which emerges in this theory is the notion of a holonomic  $D$ -module. We will study this notion in details and show that it has many non-trivial applications in analysis.

In our discussion we will try to emphasize that the construction that arise in  $D$ -module theory have very clear origin in analytic problems related to  $D$ -modules.

Later in the course I will (slowly) introduce the cohomological technique for studying  $D$ -modules. We will see that the central notion of a holonomic  $D$ -module have an equivalent description in homological terms.

### Books

"A Primer on Algebraic  $D$ -modules" by S.C.Coutinho, which gives a pretty elementary exposition of the theory.

### Home works.

Home works for this course will be given weekly. They constitute an important part of the course - many topics for which there is not enough time in the lectures will be discussed in homeworks.

Homeworks will be graded by Dmitry Gurevich (he is currently a Ph. D student at the Weizmann Institute).

Dimitry also agreed to give one hour seminar a week in which he will discuss the homeworks and also discuss other topics related to the material of the course.