# Representations of p-adic reductive groups

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In this course I will try to explain main ideas behind the local Langlands correspondence (LLC).

Last year I gave a course on representations of $p$-adic groups where I mostly described the Harish-Chandra theory how to reduce description of irreducible representations to description of cuspidal once. I will use this theory in my course, though I will repeat main definitions and results.

In this course the main attention will be given to the Langlands' picture that gives a classification of irreducible representations of a $p$-adic group in terms of Galois data for the dual group.

I will start with detailed description of the structure of $p$-adic fields and their Galois groups. I will describe the class field theory for $p$-adic fields and describe the Weil group $W(F)$ that is the "correct" version of the Galois group $Gal(F)$.

Then I will discuss LLC for the groups $GL(n, F)$.

In the second part of the course I will discuss LLC for representations of reductive groups. I will shortly recall basic definitions and properties of reductive groups and discuss in detail the notion of the dual group.

I will formulate many results without proofs, but will try to explain the ideology of the proofs and the interconnection of these results.

In this course I will mostly deal with the local situation (related to one particular prime). However I intend also to give short description of the corresponding global situations since they usually allow to understand the motivation behind the local statements.

The prerequisites for this course are

1. Familiarity with local fields
2. Standard Galois theory
3. In the second part of the course I will consider reductive groups over $p$-adic fields,

so some familiarity with the notions related to reductive groups will be quite helpful.

I will use many sources for this course. In particular

1. A.W. Knapp, Introduction to Langlands Program.

2. S. Kudla, The local Langlands Correspondence: The Non-Archimedean Case.

3. Papers from the book "An Introduction to the Langlands Program"

I will try to place copies of these texts into the course page.