Supergeometry and its applications in Physics

Course Description.

In this course I will discuss several topics related to Super Geometry.

This is an area of Geometry that arose relatively recently (around 40 years ago) motivated by developments in Physics.

The main idea of Super Geometry is to describe a "classical" picture that corresponds to fermionic particles in quantum mechanics.

It turns out that such classical picture exists - we can call it Super Mathematics. It mirrors many notions of usual Mathematics especially in such areas as linear algebra, differential geometry, algebraic (and analytic) geometry.

Many of the notions of Super Mathematics are generalizations (usually straightforward, but sometimes rather sophisticated) of notions in usual Mathematics. Some of the objects arising in Super Mathematics have been known before in some form.

There are several objects in Super Mathematics that allow to formulate and understand some known phenomena in much more natural way than it was done before. In general, though some might argue that "Super" does not add too much to the existing Mathematics, I think that to ignore this field is very much like insisting on looking at the world with just one eye.

The course will essentially consists of three parts.

In the first part I will describe "Super" analogues of many notions of classical Mathematics (Super Linear Algebra, Super Differential Geometry, Super Algebraic Geometry).

In the second part I will describe some applications of these notions to different areas in Algebra, Analysis, Geometry and Lie Theory.

In the third part I will try to describe some applications of these ideas in Physics. They mostly deal with infinite-dimensional manifolds, so I will discuss them on physical level of proofs.

In particular, if time permits, I would like to discuss the relation of these notions to general notion of Super-symmetries.

I will also try to describe super symmetric physical theories arising from super σ-models and super strings.

Books

My exposition will mostly follow the lectures in the book

"Quantum Fields and Strings: A course for Mathematicians"

published by AMS. Many of these lectures might be found on the website of the IAS (Institute for Advanced Study).

I will also use papers by Albert Schwarz who has many interesting papers on Super Geometry.