Algebraic Geometry and Commutative Algebra.

Joseph Bernstein **Fall 2011**

Course description:

This is a second part of year long basic course in Algebraic geometry for toar sheny. I complement the exposition of Algebraic Geometry by necessary facts from commutative algebra.

Books. In my exposition I mostly follow the book:

Algebraic varieties by G.R.Kempf, Cambridge University Press (London Math. Society, Lecture Notes Series, v.172).

Sometimes for exercises I will use the book **Introduction to commutative algebra** by M.F. Atiyah and I.G.MacDonald.

Home assignments. I will be giving problem assignments weekly. These problem assignments are the integral part of the course - they will contain many important points for which there is not enough time in the course itself.

The grades for home assignments will be a factor in the final grade for the course.

Exams. There will be a midterm exam in class and a final take home exam.

Syllabus of the first part of the course (Spring semester 2011).

Affine algebraic varieties
Zariski topology
Noether's normalization lemma
Hilbert's basis theorem and Nullstellensatz
Projective varieties and general algebraic varieties
Products of algebraic varieties
Separated and complete varieties
Decomposition into irreducible components
Dimension - different definitions and properties
Principal ideal theorem
Smooth points and tangent spaces
Degree of a projective variety
Classical examples of algebraic varieties
Elements of Schemes theory

Syllabus of the second part of the course (Fall semester 2011).

Recall basic notions and results from first part of the course
Algebraic curves and their non-singular models
Riemann-Roch theorem - elementary approach
Sheaves
Coherent sheaves and localization. Serre's lemma
Cohomologies and elements of homological algebra.

Higher cohomological operations with sheaves. Base change

Different versions of Riemann-Roch theorem and its applications.

Jacobians of curves

Weil's proof of Riemann hypothesis for curves over finite fields.

If time permits I will discuss the generalization of some of these notions to schemes.