

Algebra B 3 – Commutative Algebra.
Syllabus.

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Fall 2012

- (1) Rings and Algebras; polynomial algebras
- (2) Ideals, quotient rings
Fields and maximal ideals
Integral domains and prime ideals
- (3) Modules, category of modules. Monomorphisms, epimorphisms and isomorphisms.
Finitely generated modules, Nakayama Lemma, Hamilton - Cayley identity.
Finitely generated and finite algebras.
- (4) Principle ideal domains and modules over them.
- (5) Polynomial algebras, Nullstellensatz and *Specm*.
Prime ideals and *Spec*
- (6) Noetherian and Artinian rings.
Hilbert Basis Theorem
Noetherian spaces and irreducible components
- (7) Exact sequences of modules.
Localization functor and its exactness.
Localization on *SpecS*. Local algebras.
Associated primes and primary decomposition.
- (8) Tensor product of modules.
Tensor product of algebras.
Extension and restriction of scalars.
- (9) Flatness
- (10) Graded rings and modules. Completion.
Artin-Rees lemma.
Dimension theory.
- (11) Integral dependence, Normal rings
Regular rings
- (12) Homological methods and applications
- (13) Valuations, discrete valuation rings and Dedekind domains