Available online at www.sciencedirect.com
SCIENCE DIRECT®
Games and Economic Behavior 47 (2004) 172–199
GAMES and Economic Behavior
www.elsevier.com/locate/geb

Games with espionage

Eilon Solan [a,b] and Leeat Yariv [c,*]

[a] *Department of Managerial Economics and Decision Sciences, Kellogg School of Management, Northwestern University, USA*
[b] *School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israel*
[c] *Department of Economics, University of California at Los Angeles, Los Angeles, CA 90095, USA*

Received 13 May 1999

Abstract

We consider normal form games in which two players decide on their strategies before the start of play and Player 1 can purchase noisy information about his opponent's decisions concerning future response policies (i.e., *spy* on his opponent). We give a full characterization of the set of distributions over the players' payoffs that can be induced by such equilibria, as well as describe their welfare and Pareto properties. In 2×2 games we find three equilibrium phenomena: (i) when the game is non-degenerate, the information purchased is independent of its cost. The cost determines only *whether* information is purchased or not, (ii) the player who spies treats his information as if it were deterministic, even though it is correct only probabilistically, and (iii) in chain store models, espionage is used if and only if the perfect equilibrium payoff differs from the Stackelberg equilibrium payoff with Player 2 being the Stackelberg leader.

JEL classification: C72; D81; L10

Keywords: Espionage; Timing; Information pricing; Semi-correlated equilibria

"There is no place where espionage is not possible."
Sun Tzu, The Art of War, approximately 500 BC.

* Corresponding author.
E-mail addresses: eilons@post.tau.ac.il (E. Solan), lyariv@econ.ucla.edu (L. Yariv).

0899-8256/$ – see front matter
doi:10.1016/S0899-8256(03)00177-5

1. Introduction

In many real world interactions, players decide what to do long before they have to play the chosen action—an army prepares for different situations in the battlefield years before a war begins; a government decides on its policy and reactions to various scenarios before starting negotiations; an incumbent firm decides on its reactions to new market entries before entrants appear.

Once decisions are made in advance, espionage comes to mind. Suppose Players 1 and 2 engage in a two-stage sequential game that prescribes Player 1 to be the first to play an action and Player 2 to be the second. If Player 2 decides on her reactions to Player 1's move at the outset of the game, Player 1 might benefit by sending spies who will reveal the decisions made by Player 2. In fact, even if espionage is costly and provides a noisy signal of Player 2's decisions, Player 1 may still profit by utilizing it.

In essence, if it is common knowledge that Player 1 spies on Player 2, it is as if the order of actions were switched. Espionage then allows Player 2 to commit herself to an action and Player 1 to subsequently react. Thus, employment of espionage involves an interplay between the 'first mover advantage' of Player 2 and the 'second mover advantage' of Player 1.

More formally, in the present paper we study one-shot games that are extended with the ability to spy. The game is comprised of three stages. In stage 1, Player 2 chooses an action. In stage 2, Player 1 purchases an information device that reveals some information on the action chosen by Player 2. Finally, in stage 3, Player 1 chooses an action, and the original one-shot game is played. The payoff of Player 2 is her payoff in the original one-shot game, whereas the payoff of Player 1 is the difference between his payoff in the original one-shot game and the cost of the information device he purchased.

The set of available information devices, as well as their cost, are exogenous. We assume that Player 1 can always purchase a trivial device that reveals no information and costs nothing.

When concentrating on games where each player has two actions we find three phenomena that occur in equilibrium: (i) when a game is non-degenerate, the information purchased is independent of its cost. The cost determines only *whether* the information is purchased or not, (ii) the player who spies treats his information as if it were deterministic, even though it is correct only probabilistically, and (iii) in chain store models, espionage is used if and only if the perfect equilibrium payoff differs from the Stackelberg equilibrium payoff with Player 2 being the Stackelberg leader. We also study the welfare and Pareto properties of such equilibria.

The motivation for our inquiry comes from the attempt to explain the employment of different institutions providing information in a variety of economic environments. To mention a few examples, investors can employ experts who report on different attributes of firms to allow better stock investments; in certain industries, engagement in industrial espionage is common practice[1]; specialists are often hired to give forecasts before certain projects are undertaken (e.g., political advisors for defense projects).

When Player 2 knows of Player 1's espionage opportunities, Player 1's mere option to spy may in fact reduce his profit. Indeed, the option to spy can allow Player 2 to exploit a first mover advantage in the game. In such cases Player 1 would prefer not to have the ability to spy, since this ability reduces his expected payoff in equilibrium. Nonetheless, not utilizing his spying capabilities may make him even worse off, when it is impossible to commit himself not to spy.

In other words, espionage creates two opposing effects. A direct effect of espionage is the improvement of the spying party's information concerning his opponent's actions. This is, in a sense, a 'second mover advantage.' An indirect effect of espionage is driven by the opponent's knowledge that she is being spied upon. She therefore has the ability to (probabilistically) commit herself to a certain policy and take advantage of a 'first mover advantage.' This latter effect hinges on a common knowledge assumption that is crucial for the analysis and will be assumed throughout the paper. The interplay between the two forces determines the outcome of the game.

This intuitive tradeoff comes to light in the standard chain store model. There is a subgame perfect equilibrium where the Incumbent, serving as Player 2 in our framework, accommodates or fights, both with positive probability, and the Entrant, who serves as Player 1, purchases an espionage device and enters or stays out according to the signal he receives from the device. Indeed, if the Incumbent randomizes her actions, the Entrant is better off purchasing information on the Incumbent's action realization, when the cost of such information is sufficiently low. Once the Entrant employs espionage, the Incumbent can (probabilistically) commit herself to fight and best responds with a randomization between her actions.[2]

In this equilibrium the payoff of the Entrant is smaller than his payoff in the subgame perfect equilibrium of the original game, but (utilitarian) welfare increases when the cost of espionage is low enough. It turns out that, contingent on the Entrant purchasing information, the device that he purchases (i.e., the information acquired) does not depend on its cost. The cost of the device only influences the probability that the Incumbent will fight. However, if the cost of the device is too high, the Entrant will not profit by purchasing it, and there will be no equilibrium where the option to spy is used.

In the standard chain store example, Player 2 has a first mover advantage. This aspect of the game ends up playing a crucial role in the general existence of equilibria with non-trivial use of espionage. Generally, any pure perfect equilibrium in the original game is also a perfect Bayesian equilibrium in the extended game, where the players do not utilize their option of spying. Indeed, if the opponent's strategy is pure, no information

[1] From the 1997 US State Department and Canadian Security and Intelligence Service Reports, corporate espionage costs US businesses over $8.16 billion per year. Moreover, 43% of American corporations have had at least six incidents of corporate espionage.

[2] This differs from reputational explanations (see, e.g., Kreps et al., 1982; Fudenberg and Levine, 1989, 1992) both in assumptions and results. We do not assume anything about the distribution of types of Incumbents. Hence, the somewhat problematic assumption of 'irrational Incumbents' is not needed in this model. Moreover, our results predict that a non-vanishing portion of the population of Incumbents will in fact accommodate.

can be gained by way of costly espionage. Theorem 4.13 asserts that in general chain store models (corresponding to 2×2 normal form games in which one of the rows has constant payoffs), espionage is used if and only if Player 2 has the first mover advantage; that is, the perfect equilibrium of the original game differs from the Stackelberg equilibrium with Player 2 being the Stackelberg leader. We call this phenomenon *the principle of first mover*. Interestingly, the principle of first mover is in fact peculiar to chain store models and does not hold for all 2×2 games. Intuitively, the second mover advantage of Player 1 is sometimes sufficiently strong to assure the existence of equilibria with non-trivial use of espionage even when Player 2 does not have a first mover advantage.

Another interesting phenomenon is that in practically all 2×2 games (excluding degenerate cases) the cost function of information influences the decision *whether* to purchase information or not, but not *which* device to purchase. We call this phenomenon *the principle of cost-independence*.

We generalize the chain store example and characterize general chain store models for which only one player profits from the existence of espionage and such games for which both players profit from the availability of espionage. These two classes turn out to be exhaustive. We also discover that for both classes, for a sufficiently low cost of information, espionage provides an efficiency improvement.

The game structure we propose enables players to correlate their actions. Indeed, when a player receives some information on his opponent's realized action, making use of this information would imply a correlation between the players' actions. Unlike the correlated equilibria scenario, in which both players receive a signal from a third party, here only Player 2 can effectively send a signal. In Theorem 4.6 we provide a full characterization of equilibria with espionage as a modified set of the correlated equilibria of the original game.

Our model has some similarities to games with communication (see, e.g., Forges, 1986; Myerson, 1991, and the references therein). There are two main differences between the models. First, in our framework the signal is a stochastic function of Player 2's (irreversible) action, whereas in games with communication signals precede the players' action choices. Second, the scope of the noise characterizing the signal is a costly choice of Player 1, whereas in games with communication the signals are determined via cheap (in fact, free) messages that players send according to an exogenously specified communication protocol.

Nonetheless, it is worth noting that Crawford and Sobel (1982) considered a communication protocol related to the current setup. They described a sender-receiver game in which a better-informed sender sends a noisy signal to a receiver, who then chooses an action that affects the utility levels of both players. The analysis presented in this paper could be used to extend the Crawford and Sobel setup. Namely, the receiver would be allowed to *choose* the type of messages, in terms of their noisiness and corresponding cost, that the sender sends.

The literature on espionage per se appears to be very sparse. Matsui's (1989) starting point is similar to ours. He is interested in analyzing a game in which a player may receive information on her opponent's strategy and be able to subsequently revise her own choice of actions. However, Matsui approaches this general issue from a different angle than us. He considers the case of an infinitely repeated two-person game in which there is an exogenous small probability that one or both of the players will be perfectly informed of the other's supergame strategy at the outset of the game. The players have a chance to revise their strategies on the basis of this information before actual play begins. Matsui's main result is that any subgame perfect equilibrium pair of payoffs is Pareto efficient, provided that the probability of espionage is sufficiently small.

Matsui's (1989) result hinges on the fact that the *same* game is being *repeated*. This enables a player who acquires his opponent's supergame strategy to signal this information to his opponent, whereby both players switch to a Pareto efficient strategy pair. Thus, all subgame perfect equilibria entail playing a Pareto efficient strategy pair right from the outset.

In our framework, since there is only one stage, no signaling is possible. To benefit from being spied upon, Player 2 must commit herself, with positive probability, to play actions that are bad for herself (if she plays only actions that are good for herself, Player 1 can anticipate that, and does not need to purchase information). Player 2 hopes to profit by playing actions that are bad for herself, but also bad for her opponent: once Player 1 finds out that a bad action was chosen, he will play an action which is better for Player 2. This implies that a commitment to bad actions might be necessary. In particular, in contrast to Matsui's (1989) result, utilizing espionage in a one-shot game does not imply efficiency.

Another related paper is Perea y Monsuwe and Swinkels (1997). They studied a model of extensive form games, where at every information set, players can purchase a device from an information seller who is a participant in the game. The available devices differ in accuracy, and their cost is determined by the information seller. Thus, in their model, the cost of information is endogenous. Each player's purchasing decision, as well as the cost function he faces, are not revealed to the other participants of the game. Perea y Monsuwe and Swinkels are concerned with problems of evaluating information in such scenarios—what the value of the information is, how it can be computed, and how the flexibility of the information seller in setting the price of the information devices influences the play: whether it is worthwhile to set up the price in advance, or whether it is better to negotiate at every information set. Despite the underlying similarity to the model studied by Perea y Monsuwe and Swinkels (1997), our paper examines a different set of questions. We take the information seller as given and study her effects on the outcomes of the game. We concentrate on properties of equilibria from the point of view of the players and of a social planner.

Games with espionage are related to games with endogenous timing, that have been tackled with in the Industrial Organization literature. Timing of output choice in the market determines the competition structure. Sequential choice corresponds to a Stackelberg game, where the first firm to make a choice is termed the Stackelberg leader and the second is termed the Stackelberg follower. Simultaneous choice of output corresponds to a Cournot competition. Mailath (1993) allows a firm with superior information to delay its quantity decision until the decision of the less informed firm (so that decisions are made simultaneously). The unique stable equilibrium turns out to be one in which the informed firm moves first, even though the leader may earn lower ex-ante profits than it would earn if it were choosing quantities simultaneously with the follower. Sadanand and Sadanand (1996) generalized Mailath's results and showed that when there is demand uncertainty and firms endogenously choose entry timing, relative firm sizes and uncertainty jointly determine the equilibrium. Van Damme and Hurkens (1996, 1997) study the endogenous timing problem in the context of commitment. In their model, players can see the actions of players who moved before them. Thus, a player can turn the underlying simultaneous game into a sequential game in which she is the first to move. A player will then choose an action early in the game if she has a 'first mover advantage.' Our paper adds to this branch of literature in that the underlying game can be sequential and the change of turns is both probabilistic and costly. Thus, part of the optimization problem is the determination of how much resources are to be allocated to switching turns and exploiting the 'second mover advantage,' if it exists.

In our model the cost of information is exogenous. There is a vast literature dealing with the value of information. Several authors (e.g., Hirshleifer, 1971; Green and Stokey, 1981; Allen, 1986) studied the value of private information to a player. Others (e.g., Kamien et al., 1990, and the references therein) considered a situation in which an agent possesses information relevant to the players of a game in which he is not a participant. The value of information is then defined according to what this agent can achieve by behaving strategically. We view these theories as possible foundations for the cost function which we take as given.

We begin by providing the general framework for our analysis in Section 2. We then analyze a few motivating examples in Section 3. In Section 4 we study properties of espionage equilibria: we start with existence properties that hold for general $n \times m$ one-shot games in normal form in Section 4.1. We then provide a full characterization of the set of equilibria with espionage in one-shot normal form games in Section 4.2. In Sections 4.3 and 4.4 we concentrate on general 2×2 one-shot games in normal form and on chain store models. This allows us to point out some of the driving forces in the current setup. Section 5 summarizes the paper and suggests some possible avenues for future research. Technical proofs are relegated to Appendix A.

2. General framework

For every finite set K, $|K|$ is the number of elements in K, and $\Delta(K)$ is the set of probability distributions over K. For every $\mu \in \Delta(K)$, $\mu[k]$ is the probability of $k \in K$ under μ, and $\mu[K'] = \sum_{k \in K'} \mu[k]$, for every $K' \subseteq K$. We identify each $k \in K$ with the probability distribution in $\Delta(K)$ that gives unit weight to k.

2.1. The model

We consider two-player non-zero sum games in normal form. Player 1 is the row player and Player 2 is the column player. We denote by $I = \{1, \ldots, n\}$ and $J = \{1, \ldots, m\}$ the actions of the two players, and by $A = (a_{ij})$ and $B = (b_{ij})$ the two payoff matrices. The game in normal form (A, B) will be referred to as the *base game*. A *game in normal form with espionage*, or simply the *extended game*, is a tuple $G = (A, B, S, Q, \varphi)$ where

(i) (A, B) is a base game,
(ii) S is a finite set of signals,

(iii) Q is a set of functions $q : J \to \Delta(S)$,
(iv) $\varphi : Q \to \mathbf{R}$ represents the cost of information, that is, $\varphi(q)$ is the cost of information device $\Phi(q)$. We assume $\varphi \geqslant 0$.

For each $q \in Q$ there corresponds an information device $\Phi(q)$, which, when action j is chosen by Player 2, gives a (probabilistic) signal s with probability $q(j)[s]$. Note that an information device q can be represented by an $m \times |S|$ Markov matrix, in which the entry (j, s) is equal to $q(j)[s]$. In particular, Q is (equivalent to) a subset of a Euclidean space. In the sequel we identify each function $q \in Q$ with the corresponding information device $\Phi(q)$, and with the corresponding matrix. Finally, the description of the game is common knowledge.

The extended game is played as follows:

Stage 1: Player 2 chooses an action $j \in J$.
Stage 2: Player 1 purchases an espionage device $\Phi(q)$ from the set Q of available devices.
Stage 3: Player 1 receives a signal $s \in S$, where $\mathrm{Prob}(s \mid j) = q(j)[s]$.
Stage 4: Player 1 chooses an action $i \in I$.

The players' payoffs are $(a_{ij} - \varphi(q), b_{ij})$.

A pure strategy for Player 2 is a pure action $j \in J$, and a mixed strategy is a probability distribution y over J. A pure strategy for Player 1 is a pair (q, x) where $q \in Q$ is the information device he purchases in stage 2, and $x = (x(s))_{s \in S} \in I^S$ is a function that assigns a pure action to be played in stage 4 for any given signal received in stage 3. A mixed strategy for Player 1 is a probability distribution μ over $Q \times I^S$.

We denote by $\pi^l(y; \mu)$, $l = 1, 2$, the payoff to Player l when Player 2 plays the mixed strategy y, and Player 1 plays the mixed strategy μ. Formally,

$$\pi^1(y;\mu) = \sum_{(i,j,s)\in I\times J\times S} y_j \int_{(q,x)\in Q\times I^S} \big(q(j)[s]a_{ij} - \varphi(q)\big) I\big(x(s) = i\big)\,\mathrm{d}\mu, \quad \text{and}$$

$$\pi^2(y;\mu) = \sum_{(i,j,s)\in I\times J\times S} y_j \int_{(q,x)\in Q\times I^S} ql(j)[s]b_{ij} I\big(x(s) = i\big)\,\mathrm{d}\mu,$$

where $I(x(s) = i)$ is equal to 1 if $x(s) = i$, and is equal to 0 otherwise. The functions π^1 and π^2 are continuous. Moreover, π^1 is linear in μ, and π^2 is linear in y.

Definition 2.1. An information device q is *trivial* if it gives no information to Player 1; that is, $q(j)[s] = q(j')[s]$ for every $s \in S$ and every $j, j' \in J$.

We make the following assumptions on the components of the game:

A.1 Q contains a trivial device.
A.2 The cost of any trivial device is zero.
A.3 The set of available devices Q (which is equivalent to a subset of a Euclidean space) is convex and compact.
A.4 The cost function φ is continuous and convex over Q.

In some situations it is natural that the signals coincide with the actions of Player 2.

Definition 2.2. The extended game is *canonical* if $S = J$; that is, the set of signals coincides with the set of actions of Player 2.

If Player 2 has only two actions (say, Left and Right), $|J| = 2$, then a canonical device is characterized by two numbers: the probability that it reports Left when the actual action chosen by Player 2 is Left, and the probability that it reports Right when the actual action chosen by Player 2 is Right. A device in which these two probabilities are the same is called *symmetric*. Thus, a symmetric device is characterized by its accuracy: the probability with which it reports the correct action. Formally, the set of *symmetric information devices* in a 2×2 canonical game is defined by

$$Q^* = \bigl\{q : J \to \Delta(J) \bigm| q(j)[j] = q(j')[j'] \ \forall j, j' \in J\bigr\}.$$

2.2. Espionage equilibria

Definition 2.3. *Espionage equilibria* are perfect Bayesian equilibria (PBE) of the extended game. An espionage equilibrium is *true* if Player 1 purchases a costly information device with positive probability.

Note that if there is a non-trivial information device that costs nothing, then Player 1 cannot lose by purchasing it. The question is, then, whether Player 1 will also purchase a costly device.

As we see later (Theorem 4.1), assumptions **A**.3 and **A**.4 are sufficient for the extended game to admit an espionage equilibrium.

A strategy of Player 1 may involve choosing an information device from countably many, or even a continuum, of possible devices. We will be interested in those strategies in which he chooses a device from a finite set of devices.

Definition 2.4. A strategy μ of Player 1 has *finite support* if there exist $K \in \mathbf{N}$ and $q_1, \ldots, q_K \in Q$ such that $\mu[\{q_1, \ldots, q_K\} \times I^S] = 1$. The strategy is *simple* if $K = 1$; that is, $\mu[\{q\} \times I^S] = 1$ for some $q \in Q$.

If μ has finite support, so that $\mu[\{q_1, \ldots, q_K\} \times I^S] = 1$ for some $q_1, \ldots, q_K \in Q$, we define $\alpha_k = \mu[\{q_k\} \times I^S]$ to be the probability that the device q_k is chosen, and $z_k = (z_k(s))_{s \in S} \in (\Delta(I))^S$ by

$$z_k(s)[i] = \mu\bigl[\{q_k\} \times \bigl\{x : S \to I \mid x(s) = i\bigr\}\bigr] \big/ \alpha_k$$

whenever $\alpha_k > 0$. If $\alpha_k = 0$, z_k may be chosen arbitrarily. $z_k(s)[i]$ is the probability that, conditional on q_k being purchased, if the signal s is received, the action i is played by Player 1. Thus, if q_k is purchased and the signal s is received, Player 1 essentially plays the mixed action $z_k(s)$. For simplicity we write $\mu = \sum_{k=1}^{K} \alpha_k(q_k, z_k)$. If μ is simple, we write $\mu = (q, z)$.

As we prove below (Theorem 4.2), in every extended game there exists an espionage equilibrium where the strategy of Player 1 has finite support.

Consider a canonical game and a strategy $\mu = \sum_{k=1}^{K} \alpha_k(q_k, z_k)$ of Player 1 with finite support. As discussed above, since $S = J$, when Player 1 purchases a non-trivial device q_k and receives the signal j, he essentially plays the mixed action $z_k(j)$. One can then ask whether $z_k(j)$ is a best reply against j in the base game. If this is the case, Player 1 plays *as if* he completely believes the report of the device, treating it as if it were deterministic.

Definition 2.5. A strategy with finite support $\mu = \sum_{k=1}^{K} \alpha_k(q_k, z_k)$ in a canonical game has *complete belief* if for every $k = 1, \dots, K$ such that $\varphi(q_k) > 0$ *and every* $j \in J$, $z_k(j)$ is a best reply of Player 1 against j in the base game.

Since the accuracy of the signal Player 1 receives is not perfect, a best reply of Player 1 in the extended game need not have complete belief. In what follows we show that in 2×2 canonical games with symmetric information devices, every equilibrium with finite support has complete belief (see Lemma 4.8). Example 4.10 below shows that this phenomenon does not hold in general.

2.3. On the cost function

The cost function φ is a function from the set of Markov matrices to the real numbers. One might want to impose conditions on this function. For example, swapping two columns in the matrix does not change the information of Player 1 whatsoever, but changes the device we are dealing with. One would like the cost function to give the same cost to two such matrices.

We would expect that if one information device is 'more informative' in some sense than another, it should also cost at least as much. To make this idea more rigorous, we use the Blackwell (1950) partial ordering on information devices (known also as 'garbling' in the information theory literature).

Denote by $\mathcal{M}_{nm}$ the space of all $n \times m$ Markov matrices. Then $\mathcal{M}_{nm}$ is a compact convex subset of $\mathbf{R}^{nm}$.

Definition 2.6. Let $P_1, P_2 \in \mathcal{M}_{nm}$. $P_1 \succcurlyeq P_2$ if and only if there exists a Markov matrix $M \in \mathcal{M}_{mm}$ such that $P_2 = P_1 M$.

Intuitively, P_1 is defined to be at least as good as P_2 if P_2 is a noisy distortion of P_1. Alternatively, P_1 is at least as good as P_2 if a player who receives information according to P_1 can pretend to be playing according to P_2 by ignoring some of his information. In particular, Player 1 will achieve at least as high a payoff with device P_1 as with device P_2, for *any* game.

An example of a continuous and convex cost function that preserves the Blackwell relation is the following. Let Q_0 be the set of all *non-informative* $n \times m$ Markov matrices; that is,

$$Q_0 = \{q \in \mathcal{M}_{nm} \mid \text{all rows of } q \text{ are identical}\}.$$

Q_0 is a compact and convex subset of $\mathbf{R}^{nm}$, and any $q \in Q_0$ corresponds to a trivial information device.

Define a continuous function $c : \mathcal{M}_{nm} \to \mathbf{R}$ by:

$$c(q) = \operatorname{dist}(q, Q_0) = \min_{q' \in Q_0} \|q - q'\|_1,$$

where for every matrix $x = (x_{ij})$, $\|x\|_1 = \sum_{i,j} |x_{i,j}|$.

Since Q_0 is convex, c is a convex function. Moreover, c preserves the Blackwell relation. Indeed, let $q' \in Q_0$ such that $\operatorname{dist}(q, Q_0) = \|q - q'\|_1$, and denote $r = q - q'$. Then for any Markov matrix M, $q'M \in Q_0$, $\sum_{t \in S} M_{st} = 1$ for every fixed $s \in S$, and

$$\begin{aligned} c(qM) &= \operatorname{dist}(qM, Q_0) \leqslant \|qM - q'M\|_1 = \|rM\|_1 = \sum_{j,t} \left| \sum_s r_{js} M_{st} \right| \\ &\leqslant \sum_{j,t} \sum_s |r_{js}| M_{st} = \sum_s \left(\sum_j |r_{js}| \right) \left(\sum_t M_{st} \right) = \sum_{j,s} |r_{js}| = \|q - q'\|_1 \\ &= \operatorname{dist}(q, Q_0) = c(q). \end{aligned}$$

Note that for every non-negative, continuous and convex function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, the composition $f \circ c$ preserves the Blackwell relation, and is convex. Indeed, any such function f is monotonically increasing. Let $\alpha \in [0, 1]$ and $x, y \in \mathcal{M}_{nm}$. Then, from the monotonicity of f and the convexity of c and f,

$$\begin{aligned} (f \circ c)\bigl(\alpha x + (1 - \alpha) y\bigr) &\leqslant f\bigl(\alpha c(x) + (1 - \alpha) c(y)\bigr) \\ &\leqslant \alpha (f \circ c)(x) + (1 - \alpha)(f \circ c)(y), \end{aligned}$$

and therefore $f \circ c$ is convex.

3. Examples

In this section we provide several motivating examples that illustrate the main results of the paper. All the examples are of canonical games in which each player has only two possible actions, and the information devices are symmetric—they report the correct action with some fixed probability and the incorrect action otherwise. We therefore identify Q with the interval $[1/2, 1]$, where $q \in [1/2, 1]$ is the accuracy of the device $\Phi(q)$. Note that $q = 1/2$ corresponds to the trivial device, and hence $\varphi(1/2) = 0$. Moreover, $\varphi : [1/2, 1] \to [0, \infty)$ is non-decreasing.

For the examples it is convenient to assume furthermore that the cost function $\varphi(q)$ is twice differentiable and strictly convex.

We begin with studying the “Matching Pennies” game. For this game, we find the set of simple espionage equilibria for every given cost function. In particular, we identify when espionage is utilized. Moreover, we characterize the set of all distributions over the entries of the payoff matrix that can be induced by an espionage equilibrium for some cost function. This characterization is carried out for general games in Theorem 4.6.

We also provide an example where simple espionage equilibria do not exist.

We then study chain store models; we characterize when there is a true espionage equilibrium, and when this new equilibrium is more efficient.

We will see that in this case the principle of cost-independence holds: the device that is purchased in equilibrium is independent of its cost. The cost only influences the decision whether or not to purchase an information device.

Finally, we provide a game where both players benefit if Player 1 uses his ability to spy.

Example 3.1 (*Matching Pennies*). We look at the standard Matching Pennies game.

	Left	Right
Top	$1, 0$	$0, 1$
Bottom	$0, 1$	$1, 0$

If Player 2 assigns probability y^* to Left in equilibrium, Player 1 solves:

$$\max_q \bigl\{ y^* q + (1 - y^*) q - \varphi(q), \max\{y^*, 1 - y^*\} \bigr\} \\ = \max_q \bigl\{ q - \varphi(q), \max\{y^*, 1 - y^*\} \bigr\}. \qquad (1)$$

The first term in the maximization refers to the payoff achieved by purchasing information and the second term corresponds to the maximal payoff achievable without purchasing information.

We look for a true simple espionage equilibrium. Denote by $\Phi(q^*)$ the information device purchased by Player 1 in such an equilibrium (if it exists).

q^* is chosen to maximize the first term in (1). The first order condition implies that if $q^* < 1$ then $1 = \varphi'(q^*)$, and q^* depends on the cost function. In Theorem 4.11 we will see that only rarely does the information device purchased at equilibrium depend on the cost function. The 'Matching Pennies' game is such a degenerate game.

If $q^* < 1$, for a true espionage equilibrium we need $\varphi'^{-1}(1) - \varphi(\varphi'^{-1}(1)) \geqslant \max\{y^*, 1 - y^*\} \geqslant 1/2$, so that the right-hand side of (1) will be equal to $q^* - \varphi(q^*)$, and Player 1 will not benefit by not purchasing a device.

Note that for this specific game, any $y \in [1 - q^* + \varphi(q^*), q^* - \varphi(q^*)]$ is part of an equilibrium. In particular, the set of distributions over the entries of the matrix that can be induced by an espionage equilibrium is

	Left	Right
Top	yq	$(1-y)(1-q)$
Bottom	$y(1-q)$	$(1-y)q$

where $1/2 < q \leqslant 1$ and $1 - q < y < q$. In Theorem 4.6 we characterize for every one-shot game the set of distributions over the entries of the matrix that can be induced by some espionage equilibrium (without the restriction to canonical games or symmetric information devices).

Example 3.2 (*Non-existence of a simple espionage equilibrium*). Consider the following zero-sum game:

	Left	Right
Top	$1, -1$	$0, 0$
Bottom	$0, 0$	$2, -2$

This is the Matching Pennies game with different payoffs for different matchings. We claim that there is no simple espionage equilibrium in this game.

The mixed equilibrium in the base game is $((2/3, 1/3), (2/3, 1/3))$. For φ small enough (e.g., $\varphi(3/4) < 1/3$), this mixed equilibrium is no longer an equilibrium in the extended game. Suppose Player 2 plays a mixed strategy $(y, 1-y)$. If information of quality $q > 1/2$ is purchased, the payoff of Player 2 is $-yq - 2(1-y)q = yq - 2q$, which is maximized at $y = 1$. If $y = 1$ no espionage is needed, but if espionage is not used, the only possible equilibrium is the mixed equilibrium of the base game. Hence, for sufficiently low cost functions there is no simple espionage equilibrium.

The next examples are of chain store models.

Example 3.3 (*Standard chain store model*). The game is played by an Entrant and an Incumbent. The Entrant decides whether to enter the market or stay out. If the Entrant enters, the Incumbent has to decide whether to fight or accommodate. The payoffs are as given in Fig. 1, where $a > 0$, $b > 0$. The first element of any payoff pair corresponds to the Entrant's payoff and the second element corresponds to the Incumbent's payoff.

It is well known that the unique subgame perfect equilibrium is comprised of the Entrant entering and the Incumbent accommodating, whereby the equilibrium payoff is $(b, 0)$.

Suppose now that the Incumbent must decide on her reaction before the Entrant chooses whether or not to enter and that the Entrant can purchase a symmetric canonical information device. As mentioned in the Introduction, the pure subgame perfect equilibrium remains a subgame perfect equilibrium in the extended game. We now proceed to find another espionage equilibrium where the Entrant uses his ability to spy.

Suppose that in equilibrium p^* is the probability with which the Incumbent accommodates and $\Phi(q^*)$ is the information device purchased by the Entrant: the Entrant receives the correct report with probability q^*.

We will now find the exact values of p^* and q^* that constitute a true simple espionage equilibrium. As mentioned before, in 2×2 canonical games with symmetric devices, any simple equilibrium has complete belief; hence in such an equilibrium Player 1 plays a best reply in the base game for the signal he receives.

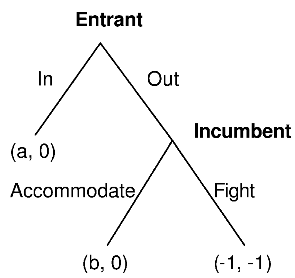

Fig. 1. A standard chain store model.

In a true simple equilibrium $0 < p^* < 1$ (else no espionage is needed). Since in equilibrium the Incumbent is indifferent between fighting, which yields $aq^* + (-1)(1 - q^*)$, and accommodating, which yields $a(1 - q^*)$, it follows that

$$q^* = \frac{1+a}{1+2a} > \frac{1}{2}. \tag{2}$$

In particular, it follows that the espionage device that is purchased by the Entrant is independent of its cost: the cost-independence principle holds. If the cost is very high, using espionage cannot be profitable to the Entrant, but for sufficiently low costs of espionage, the quality of the purchased device is determined solely by the Incumbent's payoffs.

The Entrant maximizes his expected payoff with respect to p^*. Thus, q^* is a solution of:

$$\max_q \bigl\{ p^* q b + (1 - p^*) \times \bigl(-(1-q)\bigr) - \varphi(q),\ \max\bigl\{0, p^* b - (1 - p^*)\bigr\}\bigr\}, \tag{3}$$

where the first term is his payoff if he purchases the device q, and the latter if he doesn't purchase any device. If φ is strictly convex then (3) has a unique solution. The first order condition that corresponds to the first part in (3) implies that if an espionage device $\Phi(q^*)$ is purchased then

$$(b-1)p^* = \varphi'(q^*) - 1. \tag{4}$$

Thus, if $b \neq 1$, the probability that the Incumbent fights does depend on the cost function.

To summarize, there exists a true simple espionage equilibrium if and only if p^*, q^*, and φ satisfy (2), (4), and:

$$0 < p^* < 1, \tag{5}$$

$$p^* q^* b + (1 - p^*) \times \bigl(-(1 - q^*)\bigr) - \varphi(q^*) \geqslant 0, \quad \text{and} \tag{6}$$

$$p^* q^* b + (1 - p^*) \times \bigl(-(1 - q^*)\bigr) - \varphi(q^*) \geqslant p^* b - (1 - p^*). \tag{7}$$

Observe that the following is a solution of (2), (4), and (5)–(7): $q^* = (1+a)/(1+2a)$, $p^* = 1/(b+1)$, and φ is any continuous and strictly convex function that satisfies $0 < \varphi(q^*) < b(2q^* - 1)/(b+1)$ and $\varphi'(q^*) = 2b/(b+1)$. Eqs. (6) and (7) imply that

$$\frac{1 - q^* + \varphi(q^*)}{1 - q^* + q^* b} \leqslant p^* \leqslant \frac{q^* - \varphi(q^*)}{q^* + b - q^* b}.$$

In particular, if $b = 1$ then every p^* that satisfies $1 - q^* < p^* < q^*$ is part of a solution, for an appropriately chosen cost function.

In a true simple espionage equilibrium the Entrant receives a payoff which is smaller than the payoff he receives in the perfect equilibrium of the base game. Intuitively, if the Incumbent were able to commit herself in the base game, there would be a perfect equilibrium in which the Incumbent would commit herself to fight and the Entrant would stay out. Commitment would enable the Incumbent to increase her payoff relative to the perfect equilibrium payoff she receives in the base game. The Entrant, however, would get a lower payoff when the Incumbent can commit herself to her actions. Since espionage allows the Incumbent to commit herself to her actions (albeit probabilistically), the trends in the players' payoffs are similar to those occurring when commitment tools are introduced.

Nonetheless, for certain cost functions, espionage provides an efficiency improvement. Indeed,

Claim 3.4. *There exists a cost function such that the payoffs corresponding to the true simple espionage equilibrium constitute a more efficient outcome than the payoffs corresponding to the perfect equilibrium in the base game if and only if one of the following conditions hold*:

- $b = 1$ *and* $a > 2$,
- $b \neq 1$ *and* $a > b$.

The proof of the claim appears in Appendix A.

The payoffs corresponding to equilibria with espionage are not Pareto efficient. Indeed, since the espionage devices give probabilistic signals, in a true espionage equilibrium, the payoffs $(-1, -1)$, which are Pareto inferior, are achieved with positive probability. The games considered here are not repeated (as in, e.g., Matsui, 1989); hence signaling opportunities are absent, and the main force at play is that of Player 2's ability to commit herself to her actions. Since in our model Player 2 can affect Player 1's behavior only if she commits herself to play actions that are bad both for her and for Player 1, equilibrium outcomes in the extended game may be Pareto inefficient.

Example 3.5 (*Both players profit when the Entrant uses his ability to spy*). Consider the extensive-form game depicted in Fig. 2 (we keep the notation of Entrant and Incumbent, instead of Players 1 and 2, to make the comparison with Example 3.3 more evident).

Without espionage, the unique perfect equilibrium is comprised of the Entrant staying out and the Incumbent fighting upon entrance. The corresponding payoffs are $(10, 10)$.

One can repeat the analysis performed for Example 3.3 to calculate the set of true espionage equilibria in this game. An alternative way to calculate this set is to use Theorem 4.6 below. Denote by p^* the equilibrium probability that the incumbent accommodates if the Entrant enters, and by $\Phi(q^*)$ the equilibrium device purchased by the Entrant. Setting $q^* = 2/3$, for every $1/2 < p^* < 4/5$ there exists a cost function such that p^* and q^* are the parameters that are used by the players in a true espionage equilibrium.

It is clear that both players get at least 10 in such an equilibrium (the Entrant has the alternative to stay out and get 10, while 10 is the lowest payoff in the game for

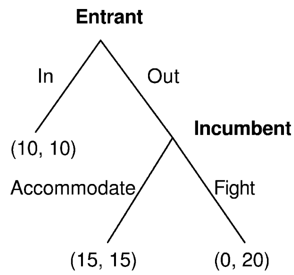

Fig. 2. A modified chain store model.

the Incumbent). Hence the ability to spy leads to a Pareto improvement over the perfect equilibrium result.

4. Properties of espionage equilibria

In this section we investigate the properties of espionage equilibria. We first prove, using standard arguments, that an espionage equilibrium always exists. We then show that there always exists an espionage equilibrium where the strategy of Player 1 has finite support.

Next, we characterize the set of all distributions over the entries of the matrix that can be induced by espionage equilibria in general normal-form games. This characterization allows us to derive two principles that hold in classes of normal-form games, namely 2×2 games and chain store models.

In Section 4.3 we concentrate on simple espionage equilibria in 2×2 canonical games. We provide a characterization of the true espionage equilibrium, identifying when it indeed exists. In particular, we establish the principle of cost-independence: if the game is non-degenerate, the device that Player 1 purchases is independent of its cost. One consequence of this characterization is that while the existence of a first mover advantage plays an important role in the current setup, it is not a sufficient proxy for the existence of a true espionage equilibrium.

In Section 4.4 we study the subclass of chain store models. For this class we deduce the principle of first mover: espionage is used if and only if the subgame perfect equilibrium in the base game is different from the Stackelberg equilibrium with Player 2 being the leader. We then characterize when the true simple espionage equilibrium is more efficient than the pure subgame perfect equilibrium in the base game.

4.1. Mixed espionage equilibrium

It is easy to see that any pure equilibrium in the game (A, B) corresponds to a pure equilibrium in the extended game, where the option to spy is not used.[3]

Theorem 4.1. *Under assumptions* **A**.3 *and* **A**.4 *the extended game admits an espionage equilibrium.*

Proof. The space of mixed strategies of Player 2 is $\Delta(J)$, which is convex and compact. By **A**.3, the space of pure strategies of Player 1 is compact. It follows that the space of mixed strategies of Player 1, which are probability measures over a compact set, is compact in the w^*-topology and, clearly, is convex. By **A**.4 the payoff function of each player is continuous, and linear in his strategy. Hence the best-reply correspondence has non-empty and convex values, and its graph is closed. By Glicksberg's (1952) generalization of Kakutani's fixed point theorem, an equilibrium in mixed strategies exists. □

[3] One class of games that has been recently studied in the literature and is comprised of games that always possess a pure equilibrium is that of potential games (see Monderer and Shapley, 1996).

Theorem 4.2. *Under assumptions* **A**.3 *and* **A**.4*, there exists an espionage equilibrium* $(y;\mu)$ *with* $\mu=\sum_{k=1}^{K}\alpha_k(q_k,x_k)$*, where* $x_k\neq x_l$ *whenever* $k\neq l$*. In particular,* μ *has finite support.*

Proof. By Theorem 4.1 there exists an espionage equilibrium $(y;\mu)$. Let $K=|I|^{|S|}=n^{|S|}$, and let $x_1,\ldots,x_K$ be *all* the functions from S to I. Let $A_k=Q\times\{x_k\}$, and $\alpha_k=\mu[A_k]$ be the probability that under μ, at stage 4, Player 1 plays according to x_k. Recall that Q is equivalent to a subset of a Euclidean space. For every k such that $\alpha_k>0$, let $q_k=\int_{A_k} q\,\mathrm{d}\mu/\alpha_k$ be the 'average' device purchased by Player 1, conditional on playing x_k. If $\alpha_k=0$, let q_k be chosen arbitrarily. Let ν be the strategy of Player 1 defined by

$$\nu=\sum_{k=1}^{K}\alpha_k(q_k,x_k),$$

that is, the device q_k is purchased with probability α_k in stage 2, and the action played in stage 4 is determined by x_k. The joint distribution on pairs (j,s), where j is an action chosen by Player 2 and s is a signal reported to Player 1, is linear in the device purchased by Player 1. Hence both μ and ν induce the same joint distribution over the space of these pairs. It follows that $\pi^2(y';\mu)=\pi^2(y',\nu)$ for every strategy y' of Player 2. By **A**.4, φ is convex, and therefore the expected cost of the device chosen by ν is at most the expected cost of the device chosen by μ. In particular, $\pi^1(y,\nu)\geqslant\pi^1(y;\mu)$. Since $(y;\mu)$ is an espionage equilibrium, $\pi^1(y;\nu)=\pi^1(y;\mu)$, and $(y;\nu)$ is an espionage equilibrium as well. □

4.2. Characterization of espionage equilibria

In this section we provide a full characterization of the set of distributions over the entries of the matrix that can arise from espionage equilibria.

Since espionage allows Player 2 to send a probabilistic signal to Player 1, it is natural to compare espionage equilibria with correlated equilibria and communication equilibria. Whereas in correlated equilibria both players receive a signal from a third party, and in communication equilibria both players send costless signals to each other according to an exogenously determined protocol, here only Player 2 can send one signal, the accuracy of which is determined by Player 1. These differences cause the set of distributions over the entries of the matrix that can arise from an espionage equilibrium to neither include, nor be included, in the set of distributions induced by correlated equilibria or by communication equilibria in the original matrix game (see, e.g., the Matching Pennies game, Example 3.1).

Nonetheless, the following example shows that sometimes espionage can be used to form a desirable correlation.

Example 4.3. Consider the following example of a 3×3 game (Moulin and Vial, 1978):

	L	M	R
T	$0,0$	$1,5$	$5,1$
I	$5,1$	$0,0$	$1,5$
B	$1,5$	$5,1$	$0,0$

The only Nash equilibrium without espionage is $\{(1/3, 1/3, 1/3), (1/3, 1/3, 1/3)\}$.

Assume that the signal space is {'Not L', 'Not M', 'Not R'}. Let q be the following device.

$$\begin{array}{lll} q(L)[\text{Not } L] = 0, & q(L)[\text{Not } M] = 1/2, & q(L)[\text{Not } R] = 1/2, \\ q(M)[\text{Not } L] = 1/2, & q(M)[\text{Not } M] = 0, & q(M)[\text{Not } R] = 1/2, \\ q(R)[\text{Not } L] = 1/2, & q(R)[\text{Not } M] = 1/2, & q(R)[\text{Not } R] = 0. \end{array}$$

This device allows Player 1 to rule out one of the actions that Player 2 did not choose. Let Q be the convex hull of q and a trivial device.

As the proof of Theorem 4.6 below shows, there is a cost function φ such that the following is an espionage equilibrium. Player 2 plays $(1/3, 1/3, 1/3)$ and Player 1 purchases the device q and plays T, I, or B, depending on whether the signal was 'Not L', 'Not M', or 'Not R', respectively. The diagonal entries are not reached in equilibrium and the corresponding payoff pair, not including the cost of espionage, is $(3, 3)$, which corresponds to the optimal correlated equilibrium of this game.

Unfortunately, the construction introduced in Example 4.3 cannot be universally replicated, as the following example illustrates.

Example 4.4. It follows from Theorem 4.6 below that no matter what the cost function is, one cannot get close to the correlated equilibrium payoff $(10/3, 10/3)$ of the following classical example (Aumann, 1974):

	Left	Right
Top	5, 1	0, 0
Bottom	4, 4	1, 5

Thus, a construction such as the one provided in Example 4.3 indeed cannot be replicated in general.

Let $I' \subseteq I$ be the set of all actions $i \in I$ that are not strictly dominated: that is, $i \in I'$ if and only if there exists $y \in \Delta(J)$ such that $\sum_{j\in J} a_{ij} y_j = \max_{i'\in I} \sum_{j\in J} a_{i'j} y_j$. In an espionage equilibrium, Player 1 will only play actions in I'.

Definition 4.5. A *semi-correlated equilibrium distribution* of a base game (A, B) is a probability distribution p over the matrix entries such that

(1) For every $i, i' \in I$, $\sum_{j\in J} p_{ij} a_{ij} \geqslant \sum_{j\in J} p_{ij} a_{i'j}$.
(2) For every $j, j' \in J$ with $\sum_{i\in I} p_{ij}, \sum_{i\in I} p_{ij'} > 0$, $\sum_{i\in I} p_{ij} b_{ij} / \sum_{i\in I} p_{ij} = \sum_{i\in I} p_{ij'} b_{ij'} / \sum_{i\in I} p_{ij'}$.
(3) For every $j, j' \in J$ with $\sum_{i\in I} p_{ij} > 0$, $\sum_{i\in I} p_{ij} b_{ij} / \sum_{i\in I} p_{ij} \geqslant \min_{i\in I'} b_{ij'}$.

Condition 1 is the standard condition of correlated equilibrium for Player 1—he cannot profit by acting as if he received a different recommended action. Condition 2 is the condition of distribution equilibrium given by Sorin (1998)—the expected payoff of Player 2 is the same, given any action she plays with positive probability. Condition 3 means that if Player 2 plays an action j with positive probability, then upon receiving a recommendation to play j, her expected payoff from playing j is at least as high as her most pessimistic payoff achieved by playing any other action j'. The most pessimistic payoff (when perfection requirements are taken into account) corresponds to the payoff achieved when Player 1 plays (spitefully) the worst action for Player 2, knowing Player 2's action, when he is restricted to undominated actions. That is, when Player 1 uses an action that he can justify to himself as a best response to some strategy of Player 2.

Each strategy pair $(y;\mu)$ in the extended game induces a probability distribution $p=(p_{ij})$ on the entries of the matrix

$$p_{ij}=\int\limits_{(q,x)\in Q\times I^S}\sum_{s\in S} y_j q(j)[s] I\bigl(x(s)=i\bigr)\,\mathrm{d}\mu,$$

where p_{ij} is the probability that under $(y;\mu)$ the entry (i,j) will be played.

We say that a probability distribution $p=(p_{ij})$ is *non-degenerate* with respect to the game (A,B) if (i) $\#\{i \mid \sum_j p_{ij}>0\}>1$ and (ii) for every $i'\in I$ there is $i\in I$ such that:

$$\sum_{j\in J} p_{ij}a_{ij} > \sum_{j\in J} p_{ij}a_{i'j}.$$

Thus, p is degenerate if either a single action of Player 1 has positive probability under p, or if Player 1 does not lose by playing any action i' regardless of the recommendation he receives.

Theorem 4.6. *If $p=(p_{ij})$ is a probability distribution induced by a true espionage equilibrium then it is a non-degenerate semi-correlated equilibrium distribution. Conversely, if p is a non-degenerate semi-correlated equilibrium distribution then there is some signal set S, some convex and compact set of information devices $Q\subseteq\{q: J\to\Delta(S)\}$, and some continuous and convex cost function $\varphi: Q\to\mathbf{R}$ such that p is the distribution induced by some true simple espionage equilibrium in the extended game (A,B,S,Q,φ).*

The result is intuitive. Player 2 chooses an action before Player 1 does, and hence must be indifferent between all actions she plays with a positive probability (condition 2 of Definition 4.5). However, she will never play an action j if all the payoffs in some other row are strictly higher than her expected payoff from playing j (condition 3 of Definition 4.5). Player 1, on the other hand, receives a signal; hence in equilibrium, he must play optimally given his signal (condition 1 of Definition 4.5). If p is degenerate then Player 1 can do just as well without purchasing a costly information device. The proof of Theorem 4.6 appears in Appendix A.

Remark 4.7. If one considers canonical symmetric devices in 2×2 games, the characterization given in Theorem 4.6 is still valid, provided one takes, instead of all semi-correlated equilibrium distributions, only semi-correlated equilibrium distributions of the form

yq	$(1-y)(1-q)$
$y(1-q)$	$(1-y)q$

4.3. 2×2 *canonical games with symmetric devices*

In this subsection we restrict ourselves to 2×2 canonical games with symmetric information devices. Thus, we identify $Q = Q^* = [1/2, 1]$.

The cost function $\varphi = \varphi(q)$ depends on a single number $1/2 \leqslant q \leqslant 1$; it satisfies $\varphi(1/2) = 0$, and is monotonically non-decreasing.

When signals and actions are binary, a device is purchased in equilibrium only if its signals are followed, in the sense that different signals lead to different actions. This is a basic observation in information economics, the idea being that if the same action is taken (even probabilistically) no matter what the signal is, there is no need for the device's reports and purchasing it is sub-optimal. In our framework, this translates formally in the following way:

Lemma 4.8. *Let $(y; \mu)$ be a true espionage equilibrium in a 2×2 canonical game with symmetric information devices, where μ has finite support. Then μ has complete belief.*

By Theorem 4.2 there exists a simple espionage equilibrium $(y; \mu)$ where $\mu = \sum_{k=1}^{K} \alpha_k(q_k, x_k)$, and $x_k \neq x_l$ whenever $k \neq l$. By Lemma 4.8, if φ is strictly convex then $\varphi(q_k) > 0$ for at most one index k. We therefore have the following:

Corollary 4.9. *In every 2×2 canonical game with symmetric information devices, if the cost function is strictly convex then there exists an espionage equilibrium $(y; \mu)$ where $\mu = \sum_{k=1}^{K} \alpha_k(q_k, x_k)$, and $\varphi(q_k) > 0$ for at most one index k.*

Thus, in 2×2 canonical games with symmetric information devices, when the information cost is strictly convex, Player 1 may use one of several costless devices, but at most one costly device.

If both players have more than two actions, Lemma 4.8 no longer holds. Player 1 may want to purchase information to differentiate between two of the actions of Player 2, but if Player 2 plays a third action, then Player 1 essentially ignores the device. This phenomenon is shown in the next example.

Example 4.10. Consider the following 3×3 game, where only the payoffs of Player 1 appear.

	L	M	R
T	3	0	0
I	0	3	0
B	0	0	1

Assume that Player 2 plays the mixed action $y = (1/3, 1/3, 1/3)$, and that Player 1 purchases the device q that with probability $1/2$ reports the action chosen by Player 2, and with probability $1/4$ reports each of the other two actions.

By an appropriate definition of the cost function, it is optimal for Player 1 to purchase q (see the proof of Theorem 4.6 for such an appropriate definition).

A simple application of Bayes' rule shows that if Player 1 receives the signal L then the probability that Player 2 actually chose L is $1/2$, and the probability that she chose each of the other two actions is $1/4$. Analogous statements hold if the signal is M or R.

Therefore, if Player 1 receives the signal L, then it is optimal for him to play T: his expected payoff is $3/2$ by playing T, $3/4$ by playing I, and $1/4$ by playing B. Similarly, if he receives the signal M, it is optimal for him to play I. However, if the signal is R, then playing B is sub-optimal: it yields him an expected payoff of $1/2$, whereas any convex combination of T and I yields $3/4$.

The following general result, which is proven in Appendix A, characterizes when a simple espionage equilibrium exists in 2×2 canonical games with symmetric information devices. Moreover, it asserts the principle of cost independence.

Denote $\alpha = a_{11} + a_{12} - a_{21} - a_{22}$ and $\beta = b_{11} + b_{12} - b_{21} - b_{22}$.

Theorem 4.11. *Let (A, B) be a 2×2 base game where no player has a weakly dominant action. Assume w.l.o.g. that $a_{11} > a_{21}$ and $a_{22} > a_{12}$.*

(i) *There exists a cost function φ for which the canonical game with symmetric information devices $(A, B, J, [1/2, 1], \varphi)$ has a true simple espionage equilibrium if and only if one of the following holds*:
   (a) $\beta = 0$ *and* $b_{12} = b_{21}$;
   (b) $\beta \neq 0$ *and* $1/2 < (b_{12} - b_{21})/\beta \leqslant 1$.

(ii) *If $\Phi(q)$ is the information device purchased by Player 1 in equilibrium, and if $\varphi(q) > 0$, then $\beta q = b_{12} - b_{21}$ [Principle of cost-independence].*

One can show that if $\alpha \neq 0$ and φ is twice differentiable, then $y = (\varphi'(q) + a_{12} - a_{22})/\alpha$ is the mixed action chosen by Player 2 in equilibrium. Since this calculation is technical and rather dull, it is omitted. Note that the Matching Pennies game (Example 3.1) satisfies $\alpha = \beta = 0$.

Remark 4.12. Theorem 4.11 proves the principle of cost-independence: if the game is not degenerate, the cost function only influences *whether* a true simple espionage equilibrium exists, but not *which* information device is purchased. The exact information device is determined *solely* by the payoff function of Player 2.

The intuition behind the principle of cost independence, as captured by the second part of Theorem 4.11, is the following. In an equilibrium, Player 2 should be indifferent between her actions. The payoff of Player 2 when she plays some pure strategy depends on (i) her payoff function, (ii) the information device purchased by Player 1, and (iii) the actions that are chosen by Player 1 given the signal he receives. However, by the principle of complete belief, Player 1's action completely depends on the signal he receives. Thus, Player 1 essentially does not control the action he plays at stage 4, and the information device is chosen to induce a proper distribution over the entries of the matrix, so that Player 2 is indifferent between her actions. Such distributions depend only on the payoffs of the base game and not on the cost function φ that is internalized by Player 1. In particular, conditional on an information device being purchased, its specifications do not depend on the cost function.

4.4. Chain store models

In this subsection we further restrict ourselves to *chain store models*; that is, 2×2 canonical games with symmetric information devices, where Player 1 has an action that yields the players the same payoff, regardless of the action of Player 2. The general game without espionage is as follows.

	Left	Right
Top	a_1, a_2	a_1, a_2
Bottom	b_1, b_2	c_1, c_2

We first characterize the conditions under which there exists a true espionage equilibrium. The theorem shows an equivalence between the existence of a first mover advantage for Player 2 and the existence of true espionage equilibria. We then characterize the conditions under which this equilibrium is more efficient than the perfect equilibrium of the base game.

The proof of Theorem 4.13, which is rather tedious, is relegated to Appendix A.

Theorem 4.13 (Principle of first mover). *Consider a chain store model in which $c_2 > b_2$. The following three statements are equivalent.*

(a) *There exists a cost function φ such that the game has a true simple espionage equilibrium.*
(b) *The perfect equilibrium of the base game is different from the Stackelberg equilibrium with Player* 2 *being the Stackelberg leader.*
(c) *Either* (i) $b_1 < a_1 < c_1$ *and* $a_2 > c_2$, *or* (ii) $c_1 < a_1 < b_1$ *and* $b_2 > a_2$.

Theorem 4.13 asserts the *principle of first mover*: in chain store models, unless Player 2 has the first mover advantage, espionage cannot be useful.

The theorem is rather intuitive. Divergence of the Stackelberg payoff from the perfect equilibrium payoff implies that Player 2 would prefer to use a reaction which is sub-optimal for her in order to get Player 1 to choose an action that differs from that prescribed by the perfect equilibrium. That is, Player 2 faces a trade-off between choosing a reaction policy that is optimal if realized (direct effect) and choosing a reaction policy that is sub-optimal, but causes Player 1 to choose a beneficial action (indirect effect). In the base game commitment is not possible and thus, according to the definition of perfect equilibrium, no player chooses an action that is sub-optimal against any tremble (in the extensive form setting this translates to sub-optimality in some decision node). However, the existence of espionage allows Player 2 to (probabilistically) commit herself to a sub-optimal reaction. Thus, as long as the costs of espionage are not extreme (low or high), espionage causes the trade-off between the direct effect and the indirect effect on Player 2's payoffs to be non-trivial.

Remark 4.14. It is important to note that the principle of first mover is specific to chain store models and does not hold in general. Indeed, one consequence of Theorem 4.11 is that, in general, first mover advantage is not the sole determinant of whether or not an equilibrium with non-trivial espionage exists. Indeed, consider the following two chicken games:

Chicken 1	Left	Right
Top	2, 5	3, 3
Bottom	1, 1	5, 2

Chicken 2	Left	Right
Top	3, 5	4, 4
Bottom	1, 1	5, 3

Both games have the same first mover advantage characteristics (if Player 2 moves first, she will get the payoff corresponding to the Nash equilibrium (Top,Left)). However,

$$\text{Chicken 1:} \quad \frac{b_{TR} - b_{BL}}{\beta} = \frac{3-1}{3+5-2-1} = \frac{2}{5} < \frac{1}{2},$$

$$\text{Chicken 2:} \quad \frac{b_{TR} - b_{BL}}{\beta} = \frac{4-1}{4+5-3-1} = \frac{3}{5} > \frac{1}{2}.$$

Hence, only the game Chicken 2 satisfies the conditions for the existence of a true simple espionage equilibrium as specified in Theorem 4.11. Conditional on Player 2 playing Left, the interests of both players are in conflict. Therefore Player 2 would be willing to play a mixed strategy only if the information structure is such that the entry (Bottom, Left) would not be reached too often. When the gap between payoffs is large, as in Chicken 1, any non-trivial device would make mixing sub-optimal for Player 2. This example illustrates the message of Theorem 4.11: Generally, the existence of espionage equilibria depends both on Player 2's first mover advantage and on Player 1's motives when the Stackelberg action is not taken.

Characterization of efficiency improvement. We now give a characterization of when the existence of espionage provides an efficiency improvement, as captured by the sum of the players' payoffs. We assume that $c_2 > b_2$.

If $b_1 < a_1 < c_1$ and $a_2 > c_2$, then the game is equivalent to the game studied in Example 3.3. In particular, Claim 3.4 characterizes when there is a more efficient equilibrium that uses espionage.

If $c_1 < a_1 < b_1$ and $b_2 > a_2$, then the game is equivalent to the one studied in Example 3.5. In particular, disregarding the cost of information, espionage provides Pareto improvement.

5. Concluding comments

In this paper we have demonstrated the effects of players' option to purchase information on their opponents' decisions (i.e., the option to spy on their opponents). This alteration of the agents' optimization problem changes the set of predictions of the game. While pure equilibria of the base game remain equilibria in the extended game with espionage, the set of mixed equilibria may change for sufficiently small costs of information. Moreover, there may be additional mixed (perfect Bayesian) equilibria when espionage is available. In general, the set of true espionage equilibria coincides with the set of non-degenerate semi-correlated equilibrium distributions.

We identified two principles that hold in various domains of 2×2 games. The *principle of first mover* asserts that in chain store models non-trivial espionage is used if and only if the perfect equilibrium of the original game does not coincide with the Stackelberg equilibrium with Player 2 being the Stackelberg leader; that is, espionage may be employed non-trivially in equilibrium if and only if Player 2 has a first mover advantage. The *principle of cost independence* claims that while the cost function of information might influence the decision whether to purchase information, it has no effect on which device is purchased in equilibrium.

Our analysis concentrated mostly on one-shot normal form games. The natural next step is to extend this study to multi-stage games with a sequence of players' decisions. This extension has economic relevance to the timing of decisions. Given that spying is possible only on policies that have already been determined, there might be a trade-off between committing oneself to policies early on in the game and waiting to a stage where the opponent's actions can be spied upon. A resolution of this trade-off can serve to determine the endogenous timing of policy decisions.

It is also worthwhile noting that espionage can potentially be considered in the context of private information that is not related to the players' actions; that is, allowing players to purchase information on others' private signals or types could extend the standard models of games with incomplete information.

Another direction for future investigation concerns the possibility of using espionage equilibria as a refinement tool. Indeed, when the cost of information is high, it is not profitable to purchase information, hence the set of equilibria of the base game coincides with the set of espionage equilibria of the extended game. When the cost of information is zero, the only equilibria of the base game that remain espionage equilibria of the extended game are the pure ones. Thus, espionage can serve as a refinement, by considering all Nash equilibria that are the least vulnerable to espionage. It is interesting to know how this type of refinement relates to existing ones (e.g. trembling hand equilibria).

Finally, our model could be extended to allow for protection against espionage (folk wisdom suggests that this phenomenon occurs in army-related enterprises, as well as in industrial/economic ones). Since espionage sometimes leads to a strict Pareto improvement, it is not clear that even if protection is very cheap, the game is equivalent to the base game. We do predict, though, that if protection is extremely costly, the game resembles the extension considered in this paper. The authors do not know how the current predictions change when protection costs are comparable to the costs of information.

Acknowledgments

We are very grateful to Drew Fudenberg for numerous helpful suggestions. We have also benefitted from conversations with Ehud Kalai, Ehud Lehrer, and Aki Matsui. We thank two anonymous referees, who found several errors in a previous version and provided many valuable suggestions.

Appendix A

Proof of Claim 3.4. The Entrant's payoff is $p^*q^*b - (1-p^*)(1-q^*) - \varphi(q^*)$ and the Incumbent's payoff is $a - q^*a$. Using (4), the sum of payoffs is

$$W = q^*\varphi'(q^*) - 1 + p^* - \varphi(q^*) + a - q^*a.$$

We look for conditions under which $W > b$.

If $b = 1$ then, by (4), $\varphi'(q^*) = 1$, and by (6) and (7), $1 - q^* + \varphi(q^*) \leqslant p^* \leqslant q^* - \varphi(q^*)$. Thus, $W > b = 1$ if and only if $q^*(2-a) - 2\varphi(q^*) - \epsilon > 2 - a$, where $\epsilon = q^* - \varphi(q^*) - p^* \geqslant 0$. This last inequality holds for some p^* and some cost function φ if and only if $a > 2$.

Assume now that $b \neq 1$ and $a > b$. Choose $\epsilon \in (0, b)$ sufficiently small so that $a - b > (1+a)(a-b+\epsilon)/(1+2a) + \epsilon(1+1/b)$. Choose $p^* > 1 - \epsilon/b$, and a cost function φ that satisfies: (i) $\varphi(q^*) < \epsilon$, and (ii) $\varphi'(q^*) = 1 - p^* + p^*b > b - \epsilon$. Then

$$\begin{aligned} W &= a + (p^* - 1) + q^*(\varphi'(q^*) - a) - \varphi(q^*) > a - \epsilon\left(1 + \frac{1}{b}\right) + q^*(b - a - \epsilon) \\ &= a - \epsilon\left(1 + \frac{1}{b}\right) + \frac{1+a}{1+2a}(b - a - \epsilon) > b, \end{aligned}$$

where the last inequality follows from the choice of ϵ.

Assume now that $b > 1$ and $a \leqslant b$. By (4) and (5), $\varphi'(q^*) < b$. In particular, $W = a + (p^* - 1) + q^*\varphi'(q^*) - q^*a - \varphi(q^*) \leqslant b$.

Finally, assume that $b < 1$ and $a \leqslant b$. By (4) and (5), $\varphi(q^*) > b$. By (4) and (2)

$$\begin{aligned} W &= a - q^*a + q^*\varphi'(q^*) + p^* - 1 - \varphi(q^*) \\ &= a - q^*a + q^*\varphi'(q^*) + \big(1 - \varphi'(q^*)\big)/(1-b) - 1 - \varphi(q^*) \\ &= \frac{a^2}{1+2a} + \varphi'(q^*)\left(\frac{1+a}{1+2a} - \frac{1}{1-b}\right) + \frac{b}{1-b} - \varphi(q^*) \\ &= b + \frac{b^2}{1-b} + \frac{a^2}{1+2a} + \varphi'(q^*)\frac{-a-b-ab}{(1+2a)(1-b)} - \varphi(q^*) \\ &< b + \frac{b^2}{1-b} + \frac{a^2}{1+2a} - b\,\frac{a+b+ab}{(1+2a)(1-b)} \\ &= b + a\,\frac{a-b}{1+2a} \leqslant b. \qquad \square \end{aligned}$$

Proof of Theorem 4.6. Let $p = (p_{ij})$ be a probability distribution induced by a true espionage equilibrium in the extended game. Since in equilibrium Player 1 plays a best response given the signal he receives, condition 1 of Definition 4.5 holds. Moreover, since in a true espionage equilibrium Player 1 purchases a costly information device, the distribution is non-degenerate. Let j and j' be such that $\sum_{i\in I} p_{ij}, \sum_{i\in I} p_{ij'} > 0$. If $\sum_{i\in I} p_{ij}b_{ij} / \sum_{i\in I} p_{ij} > \sum_{i\in I} p_{ij'}b_{ij'} / \sum_{i\in I} p_{ij'}$ then Player 2 would not play the action j' with positive probability, and so $\sum_{i\in I} p_{ij'} = 0$. In particular, condition 2 of Definition 4.5 holds for such j and j'. Now let j'' be such that $\sum_{i\in I} p_{ij}b_{ij} / \sum_{i\in I} p_{ij} <$

$\min_{i\in I'} b_{ij''}$. Player 2 cannot play j with positive probability in an equilibrium, since she gains more by playing j'', provided Player 1 plays an action in I'. However, equilibrium perfection requires that Player 1 always plays a best reply against his belief. In particular, Player 1 can only use actions in I'.

Conversely, let p be a semi-correlated equilibrium distribution. Define $y \in \Delta(J)$ by $y_j = \sum_{i\in I} p_{ij}$, and let the signal space be $S = I \cup \{\omega\}$. The signal ω will be used to force Player 1 to purchase a certain information device. For each $j \in J$ choose some $i(j)$ in $\mathrm{argmin}_{i\in I'} b_{ij}$; that is, $i(j)$ is a 'punishing action' that minimizes the payoff of Player 2 if she chooses to play j. Since $i(j) \in I'$, there exists a distribution $\hat{y}(j) \in \Delta(J)$ such that $i(j)$ is a best response against $\hat{y}(j)$. For each $j \in J$ with $y_j > 0$ define a probability distribution $x_j \in \Delta(I)$ by

$$x_j[i] = p_{ij}/y_j = p_{ij} \Big/ \sum_{i\in I} p_{ij};$$

that is, the probability induced by p on the jth column. Define the following function $q^* : J \to \Delta(S)$.

$$q^*(j) = \begin{cases} i(j) & y_j = 0, \\ x_j & y_j > 0. \end{cases}$$

The information device q^* recommends an action to Player 1: if Player 2 chooses an action j she should not play, it recommends a punishment for Player 2. Otherwise it recommends playing according to the conditional distribution given j.

Player 1's beliefs concerning his opponent's strategy are specified by the Bayesian posterior whenever defined. If the Bayesian posterior is not defined (which happens when Player 2 plays an action j with $y_j = 0$, such that $i(j)$ is not in the support of any $(x_{j'})_{j':\, y_{j'}>0}$), the belief is $\hat{y}_{j'}$, for some j' with $i(j') = i(j)$.

Define now a function $x : I \to I$ by

$$x(i) = i;$$

that is, Player 1 follows the recommendation of the device. We will now see that since p is a semi-correlated equilibrium distribution, if the players play $(y; q^*, x)$ then Player 2 cannot gain by deviating from y, and Player 1 cannot gain by deviating from x. We will then construct Q and φ that 'force' Player 1 to purchase the information device $\Phi(q^*)$.

Assume that the players play the simple espionage strategy $(y; q^*, x)$. The probability distribution induced on the pairs of actions is exactly p. By condition 2 of Definition 4.5, Player 2 is indifferent between all actions j with $y_j > 0$, and by condition 3 she cannot profit from any deviation. By condition 1, Player 1 cannot profit by not following the recommendation of q^*.

We shall now construct Q and φ that 'force' Player 1 to purchase the device $\Phi(q^*)$.

Denote by q_0 the non-informative device that sends the signal ω regardless of the action Player 2 takes. Let Q be the convex hull of q^* and q_0 (a one-dimensional space). We take the cost function to be an affine function so that for all $\alpha \in [0, 1]$, $\varphi(\alpha q^* + (1-\alpha)q_0) = \alpha\rho$, for some $\rho > 0$.

We now show that it would be optimal for Player 1 to purchase q^* and to follow its recommendation, provided ρ is sufficiently small.

Assume that Player 1 purchases the device $q = \alpha q^* + (1-\alpha)q_0$, and plays the action $z(s)$ upon receiving the signal $s \in I \cup \{\omega\}$.

Player 1's payoff is:

$$\pi^1(y; \alpha, z) = \alpha \sum_{i,j} p_{ij} a_{z(i)j} + (1-\alpha) \sum_{i,j} p_{ij} a_{z(\omega)j} - \rho\alpha.$$

Since p is a semi-correlated equilibrium distribution, $z(i) = i$ maximizes the first term. We now show that since p is non-degenerate, this quantity is maximized at $\alpha = 1$, which concludes the proof.

The function $\pi^1(y; \alpha, z)$ is linear in α; it is equal to $\sum_{i,j} p_{ij} a_{ij} - \rho$ at $\alpha = 1$, and to $\sum_{i,j} p_{ij} a_{z(\omega)j}$ at $\alpha = 0$. From the non-degeneracy condition, there exists a sufficiently low ρ for which $\sum_{i,j} p_{ij} a_{ij} - \rho > \sum_{i,j} p_{ij} a_{z(\omega)j}$. □

Proof of Theorem 4.11. We apply Theorem 4.6 in the case of canonical symmetric devices in 2×2 games. In particular, we consider probability distributions of the form

$$p_{11} = yq, \qquad p_{12} = (1-y)(1-q),$$
$$p_{21} = y(1-q), \quad \text{and} \quad p_{22} = (1-y)q. \tag{8}$$

By Theorem 4.6 we have that $qb_{11} + (1-q)b_{21} = (1-q)b_{12} + qb_{22}$, which is equivalent to $\beta q = b_{12} - b_{21}$. In particular, (ii) holds.

Since $q \in (1/2, 1]$, it follows that if a simple espionage equilibrium exists then one of the claims (i).a or (i).b holds.

For the converse, if $\beta = 0$ then (i).a implies that $b_{11} = b_{22}$ and the probability distribution (8) with $q = 1$ and any $y \in (0, 1)$ is a non-degenerate semi-correlated probability distribution. If $\beta \neq 0$, define $q_0 = (b_{12} - b_{21})/\beta$. By (i).b, $q_0 \in (1/2, 1]$. Since $a_{11} > a_{21}$ and $a_{12} < a_{22}$, there exists a unique $y_0 \in (0, 1)$ that solves the equation

$$\frac{yq_0a_{11} + (1-y)(1-q_0)a_{12}}{yq_0 + (1-y)(1-q_0)} = \frac{y(1-q_0)a_{21} + (1-y)q_0a_{22}}{y(1-q_0) + (1-y)q_0}.$$

Then the probability distribution (8) with $q = q_0$ and $y = y_0$ is a non-degenerate semi-correlated equilibrium distribution. □

Proof of Theorem 4.13. We first prove that (b) and (c) are equivalent. Since $c_2 > b_2$ action Right of Player 2 is part of any perfect equilibrium. If $a_1 < c_1$ then the perfect equilibrium is (Bottom,Right) and the Stackelberg payoff is different if and only if $a_2 > c_2$ and $b_1 < a_1$. If $a_1 > c_1$ then the perfect equilibrium is (Top,Right) and the Stackelberg payoff is different if and only if $b_2 > a_2$ and $a_1 < b_1$. To summarize, the second statement holds if and only if one of the following conditions holds:

(1) $b_1 < a_1 < c_1$ and $a_2 > c_2$; or
(2) $c_1 < a_1 < b_1$ and $b_2 > a_2$.

Note that (c) implies (a). Indeed, case (c.i) (respectively case (c.ii)) is equivalent to the chain store model studied in Example 3.3 (respectively Example 3.5).

It remains to show that (a) implies (c). We will use Theorem 4.6.

Assume there is a true espionage equilibrium, and denote the corresponding non-degenerate semi-correlated equilibrium distribution p over action combinations by

$$p_{TL} = yq, \qquad p_{TR} = (1-y)(1-q),$$
$$p_{BL} = y(1-q), \quad \text{and} \quad p_{BR} = (1-y)q,$$

where, since p is non-degenerate, $y, q \in (0,1)$. Assume w.l.o.g. that $a_2 = 0$.

Since p is non-degenerate, and by condition 1 of Definition 4.5, $\min\{b_1, c_1\} < a_1 < \max\{b_1, c_1\}$. In particular, $b_1 \neq c_1$ and $I = I'$. By Lemma 4.8, $b_1 < a_1 < c_1$ if and only if $q < 1/2$, and $c_1 < a_1 < b_1$ if and only if $q > 1/2$.

Condition 2 of Definition 4.5 guarantees that

$$(1-q)b_2 = qc_2, \tag{9}$$

and condition 3 of Definition 4.5 indicates that

$$(1-q)b_2 \geqslant \min\{0, c_2\} \quad \text{and} \quad qc_2 \geqslant \min\{0, b_2\}. \tag{10}$$

Eqs. (9) and (10), together with the assumption that $c_2 > b_2$, imply that either (i) $a_2 = 0 > c_2 > b_2$ and $q > 1/2$, or (ii) $c_2 > b_2 > a_2 = 0$ and $q < 1/2$. Thus, (c) holds. □

References

Allen, B., 1986. The demand for (differential) information. Rev. Econ. Stud. 53, 311–323.

Aumann, R.J., 1974. Subjectivity and correlation in randomized strategies. J. Math. Econ. 1, 67–96.

Blackwell, D., 1950. Equivalent comparisons of experiments. Ann. Math. Stat. 21, 265–272.

Crawford, V., Sobel, J., 1982. Strategic information transmission. Econometrica 50, 1431–1451.

Forges, F., 1986. An approach to communication equilibria. Econometrica 54, 1375–1385.

Fudenberg, D., Levine, D., 1989. Reputation and equilibrium selection in games with a patient player. Econometrica 57, 759–778.

Fudenberg, D., Levine, D., 1992. Maintaining a reputation when strategies are imperfectly observed. Rev. Econ. Stud. 59, 561–579.

Glicksberg, I.L., 1952. A further generalization of the Kakutani fixed point theorem, with application to Nash equilibrium points. Proc. Amer. Math. Soc. 3, 170–174.

Green, J.R., Stokey, N.L., 1981. The value of information in the delegation problem. HIER discussion paper No. 776. Harvard University.

Hirshleifer, J., 1971. The private and social value of information and the reward to inventive activity. Amer. Econ. Rev. 61, 561–573.

Kamien, M.I., Tauman, Y., Zamir, S., 1990. On the value of information in a strategic conflict. Games Econ. Behav. 2, 129–153.

Kreps, D., Milgrom, P., Roberts, J., Wilson, R., 1982. Rational cooperation in the finitely repeated prisoner's dilemma. J. Econ. Theory 27, 245–252. J. Econ. Theory 27, 486–502.

Mailath, G.J., 1993. Endogenous sequencing of firm decisions. J. Econ. Theory 59, 169–182.

Matsui, A., 1989. Information leakage forces cooperation. Games Econ. Behav. 1, 94–115.

Monderer, D., Shapley, L.S., 1996. Potential games. Games Econ. Behav. 14, 124–143.

Moulin, H., Vial, J.P., 1978. Strategically zero-sum games: The class whose completely mixed equilibria cannot be improved upon. Int. J. Game Theory 7, 201–221.

Myerson, R., 1991. Game Theory: Analysis of Conflict. Harvard Univ. Press.

Perea y Monsuwe, A., Swinkels, J., 1997. Selling information in extensive form games. Preprint.

Sadanand, A., Sadanand, V., 1996. Firm scale and the endogenous timing of entry: A choice between commitment and flexibility. J. Econ. Theory 70, 516–530.

Sorin, S., 1998. Distribution equilibrium I: Definitions and examples. THEMA discussion paper #9835. University Paris 10—Nanterre.
Van Damme, E., Hurkens, S., 1996. Commitment robust equilibria and endogenous timing. Games Econ. Behav. 15, 290–311.
Van Damme, E., Hurkens, S., 1997. Games with imperfectly observable commitment. Games Econ. Behav. 21, 282–308.