Uniform Equilibrium: More Than Two Players

Eilon Solan*

July 30, 1999

The basic question that this lecture is concerned about is:

Problem: Does every n-player stochastic game (with finite state and action spaces) admit a uniform equilibrium payoff?

Until this day, no counter example was found. Furthermore, we have seen that a positive answer was given for several special classes, including recursive games (Everett, 1957), zero-sum games (Mertens and Neyman, 1981), two-player absorbing games (Vrieze and Thuijsman, 1989) and two-player non zero-sum games (Vieille, 1997b). For n-player games, existence of stationary equilibrium profiles was proven for irreducible games (Sobel, 1971, Federgruen, 1978) and of 'almost' stationary equilibrium profiles for games with additive rewards and additive transitions (Thuijsman and Raghavan, 1997).

In this lecture I will review recent results for games with more than 2 players.

1 An Example

Let us begin with an example, studied by Flesch et al. (1997).

*Department of Managerial Economics and Decision Sciences, Kellogg School of Management, Northwestern University, Evanston IL 60208. e-mail: e-solan@nwu.edu. I thank Nicolas Vieille and Sylvain Sorin for their comments.

W

	L	R
T	$0,0,0$	$0,1,3$ *
B	$1,3,0$ *	$1,0,1$ *

E

	L	R
T	$3,0,1$ *	$1,1,0$ *
B	$0,1,1$ *	$0,0,0$ *

An asterisked entry means that the entry is absorbing with probability 1. The non-asterisked entry is non-absorbing. The payoff in each entry is either the non-absorbing payoff or the absorbing payoff, depending on whether the entry is non-absorbing or absorbing.

Flesch et al. proved that the game admits no stationary equilibrium (or stationary ϵ-equilibrium), and that the following cyclic strategy profile is an equilibrium:

- At the first stage, the players play $(\frac{1}{2}T + \frac{1}{2}B, L, W)$.
- At the second stage, the players play $(T, \frac{1}{2}L + \frac{1}{2}R, W)$.
- At the third stage, the players play $(T, L, \frac{1}{2}W + \frac{1}{2}E)$.
- Afterwards, the players play cyclically those three mixed-action combinations, until absorption occurs.

If the players follow this profile then their expected payoff g satisfies: $g = \frac{1}{2}(1,3,0) + \frac{1}{4}(0,1,3) + \frac{1}{8}(3,0,1) + \frac{1}{8}g$, hence $g = (1,2,1)$.

Let us verify that this profile is an equilibrium. Since the game is cyclic, the expected payoff for the players if the realized action of player 1 at the first stage is T (the continuation payoff) is $(1,1,2)$. Thus, at the first stage, player 1 is indifferent between playing T and B, player 2 receives 1/2 if he plays R and 2 if he plays L, and player 3 receives 1 if he plays E and 1 if he plays W. Thus, if everyone follows this profile from the second stage on, no one can profit by deviating at the first stage. Since both the profile and the payoffs are cyclic, similar analysis holds for all stages. Therefore, no player can profit by deviating in any finite number of stages. Since the profile is absorbing given *any* unilateral deviation, it follows that no player can profit by any type of deviation.

We shall now see a geometric presentation of this result (that was suggested by Nicolas Vieille). We are looking for an equilibrium payoff in the convex hull of $\{(1,3,0),(0,1,3),(3,0,1)\}$. In particular, it means that at most one player perturbs at every stage. The convex hull looks as follows:

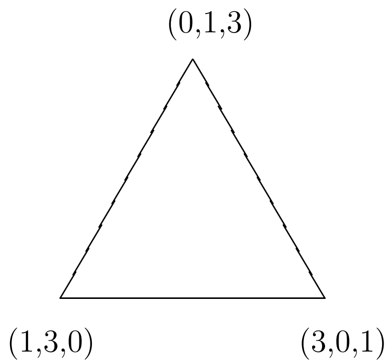

Figure 1: The Payoff Space

Let us draw the indifference lines of the players; that is, line i includes all payoffs where player i receives 1. Each indifference line divides the convex hull into two halves: the payoffs that are "good" for the player, and the payoffs that are "bad" for the player. The diagram looks as follows:

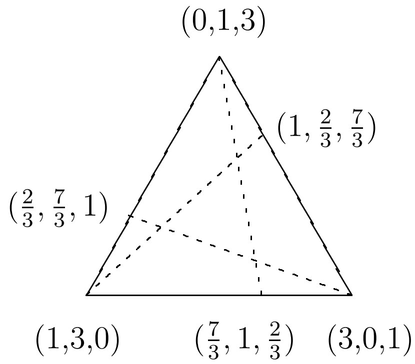

Figure 2: The Payoff Space with Indifference Lines

Assume an equilibrium profile is given, and assume that player 1 should perturb at the first stage. It means that both the equilibrium payoff and the continuation payoff are *on the indifference line of player 1.* The probability in which the first player plays B determines the distance between those two points. Similarly, if player i perturbs at stage n, then both the payoff conditional on non-absorbing before stage n and the payoff conditional on non-absorbing before stage $n+1$ are on the indifference line of player i.

Thus, the continuation payoff at stage n must be on the edges of the small triangle of Figure 2, and any point on these three edges is an equilibrium payoff (this was proven for this game by Flesch et al.). It turns out that the three extreme points of the dashed triangle in Figure 2 are $(1,2,1)$, $(1,1,2)$ and $(2,1,1)$.

For general payoff structure (with the same absorbing structure), as long as the intersection of the three "good" halves (the dashed triangle in Figure 2) is non-empty, there exists an equilibrium. Later we will see that if this intersection is empty, there exists a stationary equilibrium.

One could wonder if this argument works for four players as well. Unfortunately, the answer is negative. For four players we do not necessarily have such a cycle, and an example was given in Solan and Vieille (1998b).

2 Three Player Absorbing Games

In the present section we discuss the following generalization of the result of Flesch et al.

THEOREM 2.1 (SOLAN, 1999) *Every three-player absorbing game admits a uniform equilibrium payoff.*

Proof: The basic idea is to follow Vrieze and Thuijsman's (1989) proof for two-player absorbing game. Two difficulties that require special attention will arise.

Recall that $r^i(a)$ is the non-absorbing (daily) payoff to player i if the action profile a is played, $r^i_\star(a)$ is the absorbing payoff, and $p(a)$ is the probability that the game is absorbed if the action profile a is played.

Let v^i be the min-max value of player i. By Neyman (1988), v^i exists and is the limit of the λ-discounted min-max value of player i as λ goes to 0.

Step 1: Simple absorbing structure and low non-absorbing payoff.
We first deal with games that have the absorbing structure as in the above example (that is, each player has 2 actions, and only one cell is non-absorbing). Moreover, we assume that the non-absorbing payoffs are always below the min-max value; that is, $r^i(a) \leq v^i$ for every action profile a.

We assume that the action profile $a = (1, 1, 1)$ corresponds to the non-absorbing cell.

Let x_λ be a stationary λ-discounted equilibrium with a corresponding payoff $g_\lambda = \gamma_\lambda(x_\lambda)$. Using the algebraic approach, we assume that x_λ and g_λ are Puiseux functions of λ. Let x_0 and g_0 be the corresponding limits as $\lambda \to 0$.

Step 1a: x_0 is absorbing

As in Vrieze and Thuijsman (1989), if x_0 is absorbing, then it induces an 'almost' a stationary equilibrium, that yields a payoff g_0.

Step 1b: x_0 is non-absorbing

As for two players, for every λ we have

$$g_\lambda = p(x_\lambda)r_\star(x_\lambda) + (1 - p(x_\lambda))\left(\lambda r(x_\lambda) + (1-\lambda)g_\lambda\right).$$

Solving this equation, we get that the λ-discounted payoff is a convex combination

$$g_\lambda = \alpha_\lambda r(x_\lambda) + (1 - \alpha_\lambda)r_\star(x_\lambda)$$

where

$$\alpha_\lambda = \frac{\lambda(1 - p(x_\lambda))}{p(x_\lambda) + \lambda(1 - p(x_\lambda))}. \tag{1}$$

Taking the limit as $\lambda \to 0$ gives

$$g_0 = \alpha_0 r(x_0) + (1 - \alpha_0)\lim r_\star(x_\lambda). \tag{2}$$

If $\alpha_0 = 1$, then $g_0 = r(x_0)$. In this case, x_0 induces an 'almost' stationary equilibrium. Indeed, if player i can profit by deviating, then by continuity arguments this deviation is profitable also against x_λ^{-i} in the λ-discounted game, for λ sufficiently small. We assume now that $\alpha_0 < 1$.

Since $r^i(x_0) \le v^i = \lim_{\lambda\to 0} v^i_\lambda \le \lim_{\lambda\to 0} g^i_\lambda = g^i_0$ and $\alpha_0 < 1$, we get that

$$r(x_0) \le g_0 \le \lim_{\lambda\to 0} r_\star(x_\lambda).$$

Since x_0 is non-absorbing, $\lim r_\star(x_\lambda)$ is in the convex hull of the three entries neighboring the non-absorbing entry. In particular, the intersection in Figure 2 (the dashed triangle) is non-empty, and we can construct a cyclic equilibrium.

Step 2: General non-absorbing payoffs (with the special absorbing structure)

Note that if the payoff in the non-absorbing entry is good; that is, if $r^1(1, 1, 1) \ge$

$r^1_\star(0,1,1)$, $r^2(1,1,1) \geq r^2_\star(1,0,1)$ and $r^3(1,1,1) \geq r^3_\star(1,1,0)$ then $r(1,1,1)$ is an equilibrium payoff, that corresponds to the stationary strategy profile $(1,1,1)$.

Define an auxiliary game $\tilde{G}$ where the daily payoff for player i, if the *mixed action profile* x is played, is $\tilde{r}^i(x) = \min\{r^i(x), v^i\}$. Formally, for every stationary profile x, the λ-discounted payoff in $\tilde{G}$ is given by

$$\tilde{\gamma}^i_\lambda(x) = \lambda \mathbf{E}_x \left[\sum_{t=1}^{\infty} (1-\lambda)^{t-1} \left(\tilde{r}^i(x) 1_{t<t_\star} + r^i_\star(x) 1_{t \geq t_\star} \right) \right]$$

where $t_\star$ is the stage of absorption.

Since $\tilde{r}^i$ is continuous over the strategy space, the λ-discounted min-max value of player i in $\tilde{G}$, denoted by $\tilde{v}^i_\lambda$, exists. One can prove that the sequence $(\tilde{v}^i_\lambda)_\lambda$ converges to v^i, the min-max value of player i in the original game.

The function $\tilde{r}^i$ is quasi-concave and continuous, hence $\tilde{G}$ has a λ-discounted stationary equilibrium. The function $\tilde{r}$ is semi-algebraic, hence one can choose for every λ a λ-discounted stationary equilibria x_λ in $\tilde{G}$ such that the mapping $\lambda \mapsto x_\lambda$ is a Puiseux function.

We now repeat the same analysis as in Step 1b. Denote $g_\lambda = \tilde{\gamma}^i_\lambda(x_\lambda)$, and let $x_0 = \lim x_\lambda$ and $g_0 = \lim g_\lambda$.

If x_0 is absorbing then, as before, it induces an 'almost' stationary equilibrium. Thus, we assume that x_0 is non-absorbing. Then

$$g_0 = \alpha_0 \tilde{r}(x_0) + (1-\alpha_0) \lim r_\star(x_\lambda). \qquad (3)$$

where $\alpha_0 = \lim \alpha_\lambda$ and α_λ is given in (1). If $\alpha_0 = 1$ then $g_0 = \tilde{r}(x_0) \leq r(x_0)$, and as before supplementing x_0 with threat strategies yields that $r(x_0)$ is a uniform equilibrium payoff. Otherwise, $\tilde{r}(x_0) \leq v = \lim_{\lambda\to 0} v_\lambda \leq \lim_{\lambda \to 0} g_\lambda = g_0$ and therefore $g_0 \leq \lim r_\star(x_\lambda)$. Thus, the intersection in Figure 2 is non-empty, and there exists a cyclic equilibrium.

Step 3: General 3-player absorbing game

It is convinient here to view the absorbing game as a stochastic game with initial state z_0, such that all other states are absorbing.

We define the auxiliary game as in Step 2, and choose a Puiseux function $\lambda \mapsto x_\lambda$, where x_λ is a stationary equilibrium in the game $\tilde{G}$. We define $x_0 = \lim_\lambda x_\lambda$ and $g_0 = \lim \gamma_\lambda(x_\lambda)$. Equality (3) holds in this more general setup.

As before, if x_0 is absorbing then it induces an 'almost' stationary equilibrium that yields the players an expected payoff g_0. Assume now that x_0 is non-absorbing. If $\alpha_0 = 1$ then as before $r(x_0)$ is an equilibrium payoff.

Since x_0 is non-absorbing, the non-absorbing state forms a weak communicating set under x_0. In the talk "General Tools - Perturbations and Markov Chains" we have defined exit distributions from communicating sets. Let $\mathcal{Q}(x_0)$ be the set of exit distributions. We denote by $\mathcal{Q}_i(x_0)$ the set of unilateral exits of player i (previously we had only two players) and $\mathcal{Q}_0(x_0)$ the set of joint exits (exits that require perturbations of at least two players). Any exit $Q = \sum_{l \in L} \eta_l P_l \in \mathcal{Q}(x_0)$ can be decomposed (not necessarily uniquely) to a sum $Q = \sum_i \sum_{l \in L_i} \eta_l P_l + \sum_{l \in L_0} \eta_l P_l$, where $P_l \in Q_i(x_0)$ for $l \in L_i$, $i = 0, 1, 2, 3$.

Since the setup is of absorbing games, an exit yields a terminal payoff. Define the set of *terminal payoffs* w.r.t. x_0 by:

$$\mathcal{E}(x_0) = \{P r_\star \mid P \in \mathcal{Q}(x_0)\}.$$

It is easy to see that

$$\lim r_\star(x_\lambda) \in \mathcal{E}(x_0).$$

Lemma 5.2 from "General Tools - Perturbations and Markov Chains" translates to:

LEMMA 2.2 *Let $Q = \sum_l \eta_l P_l \in \mathcal{Q}(x_0)$ with a decomposition (L_i), and $g = \sum_l \eta_l P_l r_\star$. Let $\gamma : S \to \mathbf{R}^N$ coincide with $r_\star$ on $S \setminus z_0$ and $\gamma(z_0) = g$. If the following conditions hold*

1. *For every $l \in L_i$, $P_l \gamma^i_\star = g^i$.*
2. *For every player i and action a^i, $p(\cdot \mid x_0^{-i}, a^i)\gamma^i \leq g^i$.*

then g is an equilibrium payoff.

The last step of the proof is a geometric lemma that shows that if there is no cyclic equilibrium, then there exists $Q \in \mathcal{Q}(x_0)$ that satisfies Lemma 2.2. ∎

The same technique, without the need of the geometric lemma, proves the following result. An absorbing game is a *team game* if the players are divided into two teams, and the players in each team have the same payoffs (both absorbing and non-absorbing).

THEOREM 2.3 (SOLAN, 1997) *Every absorbing team game admits a uniform equilibrium payoff.*

DEFINITION 2.4 *A strategy profile σ is* (x, ϵ)-perturbed *if it has the following structure:*

1. *Any player is checked by a statistical test.*
2. *As long as no player failed the statistical test, the mixed action profile prescribed by σ is ϵ-close to x (in the supremum topology).*
3. *The first player who fails the statistical test is punished with an ϵ-min-max profile forever.*

An ϵ-equilibrium profile σ is perturbed *if there exists a stationary profile x such that σ is (x, ϵ)-perturbed.*

In all classes of non zero-sum stochastic games we have seen so far where the uniform equilibrium is known to exist, there are ϵ-equilibrium profiles that are perturbed.

The importance of having a perturbed equilibrium is the method of the proof. Most of the proofs we have seen take a sequence of stationary equilibria in ϵ-approximating games that converge when ϵ goes to 0. Mertens and Neyman (1981), Vrieze and Thuijsman (1989) and Vieille (1997a) consider the discounted game, Solan (1999) considers the discounted version of a variation of the game, and Vieille (1997b), Flesch et al. (1996) and Solan (1998a) consider approximating games where the players have constrained strategy spaces. In the ϵ-equilibrium profile the players play mainly the limit stationary strategy, and perturb to other actions with small probability. In particular, the ϵ-equilibrium profile is perturbed.

Solan and Vieille (1998b) constructed a four-player game that has no perturbed equilibrium payoff. In particular, this example hints that the classical approach may not work in general.

3 Quitting Games

We should look for a new approach to deal with n-player games. We consider the simplest class of games we can imagine — quitting games. A *quitting* *game* is an absorbing game where each player has 2 actions: *continue* and *quit*. If everyone continue then the game continues (terminates with probability 0) and the daily payoff is 0. If at least one player quits, the game terminates with probability 1. The three-player game we studied in section 1 is a quitting game.

THEOREM 3.1 (SOLAN AND VIEILLE, 1998B) *Every quitting game that satisfies the following two conditions*

1. *If a single player quits, he receives 1.*

2. *If player i quits with some other players, he receives at most 1.*

admits a subgame perfect ϵ-equilibrium payoff. Moreover, there is a cyclic ϵ-equilibrium strategy profile, but the length of the cycle can depend on ϵ.

Proof: The approach that is taken in the proof of Theorem 3.1 is different then the classical one. Instead of defining the best reply correspondence, and look for a fixed point, we look for a sequence $g_1, g_2, \ldots$ of payoff vectors such that g_k is an equilibrium payoff in the one-shot game with continuation payoff g_{k+1}. Denote by x_k the corresponding equilibrium strategy profile in this one-shot game. One can verify that if such a sequence exists, and if the sequence $(x_1, x_2, \ldots)$ is terminating with probability 1, then the profile $(x_1, x_2, \ldots)$ is an equilibrium of the quitting game.

Since it might be the case that the iterates of the best reply correspondence converge to the "all continue" mixed action, the existence of such a profile is not clear.

For every payoff vector $w \in \mathbf{R}^N$, let $G(w)$ be the one-shot game with continuation payoff w. Let

$$W = \{w \in [-\rho, \rho]^N \mid w^i \leq 1 \text{ for at least one } i\}$$

where ρ is the maximal payoff in the game (in absolute values).

We denote by $\langle G(w), x\rangle^i$ the payoff of player i in this one shot game if the mixed action profile x is played.

DEFINITION 3.2 *The action a^i is an ϵ-*best reply *against x^{-i} if*

$$\langle G(w), x^{-i}, a^i\rangle \geq \max_{b^i} \langle G(w), x^{-i}, b^i\rangle - \epsilon.$$

*The mixed action profile x is an ϵ-*equilbirium *of $G(w)$ if for every i and every $a^i \in \text{supp}(x^i)$, a^i is an ϵ-best reply against x^{-i}.*

Lemma 3.3 *For every $w \in W$, the game $G(w)$ possesses a $\rho\epsilon$-equilibrium that is absorbing with probability of at least ϵ.*

Proof: Let x be an equilibrium payoff in $G(w)$. If x prescribes all players to continue, then the corresponding equilibrium payoff is w. In particular, there exists a player i with $w^i \leq 1$, and by the first condition $w^i = 1$. Otherwise, by the second condition, there exists a player i who plays in x a fully mixed action, and is indifferent between his two actions. Let $y^j = x^j$ for all $j \neq i$, and $y^i = x^i + \epsilon$ otherwise (increase the probability to quit by ϵ). It is easy to verify that y is a $\rho\epsilon$-equilibrium that is absorbing with probability of at least ϵ. ■

Define the correspondence $\phi : W \to W$ as follows. For every $w \in W$, $\phi(w)$ is the set of all $\rho\epsilon$-equilibria that are absorbing with probability of at least ϵ. This correspondence has non-empty values, and clearly is upper-semi-continuous.

Lemma 3.4 *For every upper-semi-continuous correspondence ϕ with non-empty values from a compact set W into itself there exists a sequence $w_1, w_2, \ldots$ such that for each i, $w_i \in \phi(w_{i+1})$.*

It is clear that one can generate a sequence $w_1, w_2, \ldots$ such that $w_{i+1} \in \phi(w_i)$.

Proof: Define $W_0 = W$ and $W_{i+1} = \phi(W_i)$. Since W is compact and ϕ upper-semi-continuous with non-empty values, W_i is compact. By induction, $W_{i+1} \subseteq W_i$, hence $W_\infty = \cap W_i \neq \emptyset$.

Let $w_1 \in W_\infty$. For each i choose a sequence $w_1 = w_i^1, w_i^2, \ldots, w_i^i$ such that $w_i^j \in \phi(w_i^{j+1})$. By taking a subsequence, assume that $w_\infty^j = \lim_i w_i^j$ exists for all j. By upper-semi-continuity, the sequence $(w_\infty^j)_j$ satisfies the lemma. ■

We have generated a sequence of continuation payoffs (w_i) such that w_i is a $\rho\epsilon$-equilibrium in $G(w_{i+1})$. Let x_i be the corresponding mixed-action profile. The profile $\mathbf{x} = (x_1, x_2, \ldots)$ is our natural candidate to be an ϵ'-equilibrium in the quitting game. However, if players follow $\mathbf{x}$ then at *every* stage each player may profit $\rho\epsilon$ by deviating. How do we know that these small profits do not aggregate?

One can now prove that either $(x_1, x_2, \ldots)$ is an ϵ'-equilibrium, or there exists a player i such that if i quits alone, everyone get at least 1, hence a stationary ϵ-equilibrium exists. ■

Problem: Can one bound the length of the cycle?
Problem: Does the result hold for general quitting games (without the two assumptions)?

4 Correlated Equilibrium

Correlation devices were introduced by Aumann (1974, 1987). A correlation device chooses for every player a private signal before the start of play, and sends to each player the signal chosen for him. Each player can base his choice of an action on the private signal that he has received.

For multi-stage games, various generalizations of correlation devices have been introduced. The most general device receives at every stage some private message from each player and has perfect recall (*communication device*, Forges (1986, 1988), Myerson (1986), Mertens (1994)). The most restrictive device bases its choice only on the currect state (and not even on past signals) (Nowak and Raghavan (1992)). In between there are devices that base their choice on past signals that were sent, but not on past play (*autonomous correlation devices*, Forges (1986)).

Nowak and Raghavan (1992) proved that in discounted stochastic games with general setup (the only restricting condition is that the transition law is absolutely continuous w.r.t. some fixed probability distribution and the Radon-Nikodym derivative satisfy some continuity condition) admits a stationary correlated equilibrium. Their use of the correlation device was to convexify the set of Nash equilibria. They define for every continuation payoff function the convex hull of the set of selections of Nash equilibria in the corresponding one-shot game (which is a subset of the set of selections of correlated equilibria). They prove that this correspondence is upper-semi-continuous and has non-empty values, and apply Kakutani's fixed point theorem.

This approach fails for undiscounted games from the same reason that the proof of existence of Nash equilibrium fails: keeping your continuation payoff high does not mean that you will eventually get this payoff.

Here we concentrate on two types of correlation devices: (i) stationary devices, that choose at every stage a signal according to the same probability distribution, independent of any data, and (ii) autonomous devices, that base their choice of new signal on the previous signal, but not on any other information.

THEOREM 4.1 (SOLAN AND VIELLE, 1998A) *Every n-player stochastic game has a correlated equilibrium, using an autonomous correlation device. The equilibrium path is sustained using threat strategies, but punishment occurs only if a player disobeys the recommendation of the device.*

A stronger result is possible for positive recursive games (those are recursive games where the payoff in absorbing states is non-negative for all players).

THEOREM 4.2 (SOLAN AND VIELLE, 1998A) *If the game is positive recursive, then the correlation device can be taken to be stationary, and deviators are punished with the min-max value.*

The proofs utilize various methods that we have already seen, and one new idea. They are divided into two steps. First we construct a "good" strategy profile; meaning, a strategy profile that yields all players a high payoff, and no player can profit by a unilateral deviation that is followed by an indefinite punishment (where in Theorem 4.1 punishment is given by the max-min value, and in Theorem 4.2 by the min-max value).

The construction of the "good" strategy profile uses the method of Mertens and Neyman (1981) for Theorem 4.1, and a variant of the method of Vieille (1997b) for Theorem 4.2.

Second, we follow Solan (1998b) and define a correlation device that *mimics* that strategy profile: the device chooses a pure action profile according to the probability distribution given by the strategy profile, and recommends each player to play "his" action in the action profile. To make deviations non-profitable, the device reveals to all players what were his recommendations in the previous stage. This way, a deviation is detected immediately, and can be punished. In particular, the device that we construct is not canonical (Forges (1988)).

We shall now explain the construction in more details. Assume for a moment that the device can base its choice of a signal on the state of the world as well as on previous signals. First we construct a device that mimics a given strategy profile σ. We will then add to it a component that prevents profitable deviations.

The device sends a recommended action to every player at each stage. If the stream of states is $(s_1, \ldots, s_n)$ and the previous recommendations were $(a_1, \ldots, a_{n-1})$, the recommendation at stage n is an action profile that is chosen according to the probability distribution $\sigma(s_1, a_1, s_2, \ldots, a_{n-1}, s_n)$.

To prevent profitable deviations, the device sends at stage n to each player both the recommended action for that stage *and* the recommended action profile of the previous stage. This way, any deviation is detected immediately, the deviator is identified by everyone, and can be punished.

Note that if punishment is given by the max-min value, the players need to correlate also in the punishment phase. In such a case, before the start of play the device chooses for every player i a sequence of i.i.d. uniformaly distributed numbers in the unit interval, and sends those numbers to all players *except* player i. If player i ever deviates, these numbers are used by all other players to correlate punishment.

To overcome the need of knowing the current state, the device sends a *vector* of signals, one for each possible stream of states. The players, who know what is the realized stream of states, pick the correct signal from the vector, and play according to it.

Problem: Does any stochastic game (*resp.* positive recursive game) admits a correlated equilibrium payoff (that is, the device sends only one signal before the start of play).

A first step to answer this question was given by Solan and Vohra (1999) for quitting games.

References

[1] Aumann R.J. (1974) Subjectivity and Correlation in Randomized Strategies, *J. Math. Econ.*, **1**, 67-96

[2] Aumann R.J. (1987) Correlated Equilibrium as an Expression of Bayesian Rationality, *Econometrica*, **55**, 1-18

[3] Everett H. (1957) Recursive Games, *Contributions to the theory of Games*, **3**, 47-78, Princeton N.J., Annals of Mathematical Studies, **39**, Princeton University Press.

[4] Federgruen A. (1978) On N-Person Stochastic Games with Denumerable State Space, *Adv. Appl. Prob.*, **10**, 452-471

[5] Flesch J., Thuijsman F. and Vrieze K. (1996) Recursive repeated games with absorbing states, *Math. Oper. Res.*, **21**, 1016-1022

[6] Flesch J., Thuijsman F. and Vrieze K. (1997) Cyclic Markov Equilibria in Stochastic Games, *Int. J. Game Th.*, **26**, 303-314

[7] Forges F. (1986) An Approach to Communication Equilibria, *Econometrica*, **54**, 1375-1385

[8] Forges F. (1988) Communication Equilibria in Repeated Games with Incomplete Information, *Math. Oper. Res.*, **13**, 2, 77-117

[9] Mertens J.F. (1994) Correlated and Communication Equilibria, In *Game Theoretic Methods in General Equilibrium Analysis*, Mertens J.F. and Sorin S. (eds.), Kluwer Academic Press

[10] Mertens J.F. and Neyman A. (1981) Stochastic Games, *Int. J. Game Th.*, **10**, 53-66

[11] Myerson R.B. (1986) Multistage Games with Communication, *Econometrica*, **54**, 323-358

[12] Neyman A. (2000) The Value of Stochastic Games (**Check Name**), These Proceedings

[13] Nowak A.S. and Raghavan T.E.S. (1992) Existence of Stationary Equilibria with Symmetric Information for Discounted Stochastic Games, *Math. Oper. Res.*, **17**, 519-526

[14] Sobel M.J. (1971) Noncooperative Stochastic Games, *Ann. Math. Stat.*, **42**, No. 6, 1930-1935

[15] Solan E. (1999) Three-Person Absorbing Games, *Math. Oper. Res.*, **24**(3)

[16] Solan E. (1997) Absorbing Team Games, *Games Econ. Behavior*, *forthcoming*

[17] Solan E. (1998a) Stochastic Games with Two Non-Absorbing States, D.P. #175, Center for Rationality and Interactive Decision Theory, the Hebrew University, Jerusalem

[18] Solan E. (1998b) Extensive-Form Correlated Equilibria, , D.P. #175, Center for Rationality and Interactive Decision Theory, the Hebrew University, Jerusalem

[19] Solan E. and Vieille N. (1998a), Correlated Equilibrium in Stochastic Games, D.P. # 1226, The Center for Mathematical Studies in Economics and Management Science, Northwestern University *and* Cahiers du CEREMADE #9848

[20] Solan E. and Vieille N. (1998b), Quitting Games, D.P. # 1227, The Center for Mathematical Studies in Economics and Management Science, Northwestern University*and* Cahiers du CEREMADE #9846

[21] Solan E. and Vohra R. (1999), Correlated Equilibrium in Quitting Games, *preprint*

[22] Thuijsman F. and Raghavan T.E.S. (1997) Perfect Information Stochastic Games and Related Classes, *Int. J. Game Theory*, **26**, 403-408

[23] Vieille N. (1997a) Equilibrium in 2-Person Stochastic Games I: A Reduction, CEREMADE D.P. #9745

[24] Vieille N. (1997b) Equilibrium in 2-Person Stochastic Games II: The Case of Recursive Games, CEREMADE D.P. #9747

[25] Vrieze O.J. and Thuijsman F. (1989) On Equilibria in Repeated Games With Absorbing States, *Int. J. Game Th.*, **18**, 293-310