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Sequential rent seeking *

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Abstract. We consider n firms which choose rent-seeking expenditures sequentially, each player anticipating the rent-seeking expenditures that will be made by later movers. We find that the earlier movers need not make larger profits than later movers, and that aggregate profits are lower than in a game in which firms make simultaneous moves.

1. Introduction

The usual analysis of rent seeking considers a Nash game – firms make their expenditures simultaneously, with no firm able to commit to an expenditure level. This paper looks instead at a Stackelberg (sequential) game – firm A makes a payment, then firm B does, then firm C, and so on. Each firm takes into account how its expenditure will affect the choices of later firms.

Our analysis can be applied at two levels. First, given institutions which allow Stackelberg behavior (e.g. a Federal Election Commission which quickly makes contributions public) would any firm want to move first? Our analysis shows that under most conditions the first mover earns higher profits than later movers, thus making Stackelberg behavior plausible. Second, which game would firms or politicians prefer? Our numerical results show that aggregate rent-seeking payments are at least as high in a Stackelberg game as in a Nash game. Therefore, politicians who receive the contributions would prefer institutions that foster a Stackelberg game, while firms would prefer the opposite.

Our results can also be used to find the conditions under which rent seeking will be extensive. For example, we shall find that in a Stackelberg game the first mover spends the most, while later movers spend and profit little. In the presence of any fixed participation costs (such as the cost of opening a lobbying office in Washington) we may therefore observe very few firms attempting to earn rents.

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2. Literature review

Several papers have considered limited forms of sequential expenditures. Linster (1993) considers a game with only two (asymmetric) players. His derivation can be simplified to show that with two players the Stackelberg and Nash solutions are identical. Dixit (1987) makes that result explicit, in a model more general than that usually considered in the rent-seeking literature. Dixit also considers a game with one leader and multiple followers who play a Nash game. Our paper is simpler by considering a game in which all firms have the same profit function. But our paper is more complicated by considering a sequence of payments, rather than a game in which all but one firms are followers.

Leininger (1993), building on work by Hamilton and Slutsky (1990), extends the standard rent-seeking game by allowing each of two players to choose whether to move first or second; when the players have different valuations of the prize, the equilibrium will have moves in a particular order.

Sequential games have been studied in the Industrial Organization literature, examining the behavior of oligopolists. But the problem there can be simplified by assuming a linear demand curve, which effectively allows each firm to ignore how its choice of output affects choices made by later movers (see Anderson and Engers, 1992).

3. The Nash model

To put our model in context, consider first the classic rent-seeking game, which assumes Nash behavior. Let n firms compete for a prize of value 1. Let firm i's rent-seeking expenditures be y_i. We follow the conventional assumption that a firm's probability of winning the prize equals the proportion of its rent-seeking expenditures to total rent-seeking expenditures. Thus, firm i's expected net benefits are $y_i/Y - y_i$, where $Y = \sum_{i=1}^{n} y_i$.

In a Nash equilibrium, firm i chooses y_i to maximize its expected net benefits, so that y_i must satisfy the first-order condition $\frac{1}{Y} - \frac{y_i}{Y^2} = 1$ or

$$y_i = Y(1 - Y) \quad i = 1, ..., n. \tag{1}$$

Summing over i yields that $Y = nY(1 - Y)$ or

$$Y = \frac{n-1}{n}.$$

The amount spent by each firm is

$$y_i = \frac{n-1}{n^2} \quad i = 1, ..., n.$$

Aggregate profits are then $1 - Y = 1/n$, with each firm's profits $1/n^2$.

4. The Stackelberg model

In our sequential model, the order of moves is fixed. Firms may enter in sequence because some firms are aware of a profitable market before others, or because some take longer to tool up. We shall take the order as exogenous, though it can of course be considered the outcome of an endogenous game.

As Dixit (1987) notes, many equilibria are possible. We shall consider only subgame perfect ones. For example, we do not allow firm 2 to threaten that it will spend a million dollars if firm 1 spends anything at all.

To see the nature of the Stackelberg solution, and the difficulties of obtaining an analytic solution, consider three firms, with firm 1 moving first, firm 2 moving second, and firm 3 third. Let y_i be firm i's expenditures. As in the standard model of rent seeking, suppose firm i's probability of winning the prize is $y_i/\Sigma_{j=1}^{n} y_j$; its expected profits are $y_i/\Sigma_{j=1}^{n} y_j - y_i$. The solution is obtained by working backwards. Firm 3 observes the sum of spending by firms 1 and 2, $g_2 = y_1 + y_2$. Maximizing profits, $y_3/(g_2 + y_3) - y_3$, requires that

$$y_3 = \sqrt{g_2} - g_2$$

. Consider next firm 2. It observes y_1, which it views as fixed. It sees y_3 as a function of y_2. Firm 2 thus maximizes

$$\frac{y_2}{y_1 + y_2 + y_3(y_2)} - y_2 = \frac{y_2}{\sqrt{g_2}} - y_2.$$

Solving the first-order condition gives

$$y_2 = 2(g_2 - g_2\sqrt{g_2}).$$

Consider now the decision of firm 1. Total rent-seeking expenditures are $g_2 + y_3 = \sqrt{g_2}$. It thus maximizes $y_1/\sqrt{g_2} - y_1$. Making use of the conditions $y_1 = g_2 - y_2$ and $y_2 = 2(g_2 - g_2\sqrt{g_2})$, the first mover's objective can be rewritten as $3g_2 - \sqrt{g_2} - 2g_2\sqrt{g_2}$. Solving the first-order condition we obtain the solutions $g_2 = (3 \pm \sqrt{3})/6$. The two solutions are 0.622 and 0.0045. Only 0.622 generates a non-negative value for y_1. The unique solution is therefore $y_1 = 0.359$, $y_2 = 0.263$, and $y_3 = 0.167$.

For more than three firms we determine the optimal solutions numerically, as explained in the next section.

4.1. *Solution of Stackelberg game*

To determine the optimal solution for each player we use a grid search. That is, instead of treating y_i as a continuous variable, we let it take discrete value, $y_i \in M$ where, for example, $M = \{0, 0.001, 0.002, ..., 1\}$.

We index the players in the order in which they move: player 1 moves first, then player 2, and so on. When player i moves, it knows how much all players preceding it spent. Since player i's payoff depends on the sum of all other expenditures, and not on the distribution of that sum across different players, we can summarize the situation player i faces by the sum of expenditures by players $1, ..., i-1$; call this g_{i-1}. The decision of player i also depends on expected spending by its followers, players $i+1, ..., n$. This expected spending can be determined as a function of the sum $g_{i-1} + y_i$. To solve the model we define the following functions:

$h_i(g) -$ total spending by players i, ..., n if players 1, .., $i-1$ spent in total g;

$y_i(g) -$ spending by player i if players $1, ..., i-1$ spent in total g.

The model can now be solved through the following relations:

1. $y_i(g_{i-1})$ is the value of y which maximizes
$$\frac{y}{g_{i-1} + y + h_{i+1}(g_{i-1} + y)} - y.$$
2. $h_{i+1} = 0$ and for $i \geq n$
$h_i(g_{i-1}) = y_i(g_{i-i}) + h_{i+1}(g_{i-1} + y_i(g_{i-1}))$.

We start by computing, for each possible value of g_{n-1}, the value of $y_n(g_{n-1})$ and then the value of $h_n(g_{n-1}) = y_n(g_{n-1})$. We continue by computing $y_{n-1}(g_{n-2})$ and then $h_{n-1}(g_{n-2})$ for every possible value of g_{n-2}. Computations continue in decreasing order of indices until we have the function $h_2(g_1)$, which allows computing $y_1(0)$. We compute the solutions for $y_2, ..., y_n$ by computing $g_i = g_{i-1} + y_i$ and $y_i = y_i(g_{i-1})$.

For some values of g_{i-1} several values of y_i may give the same, maximum, profits to player i. In the following we suppose that the player chooses the largest y_i among these values. That essentially supposes each preceding player considers the worst that can happen to it (where larger payments by successors reduce the profits of a predecessor). We also tried the opposite assumption, that a player chooses the smallest of several optimal payments, and found no difference in the pattern of results.

The table below shows the optimal response of the last player, $y_n(g_{n-1})$.

g_{n-1}	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y_n(g_{n-1})$	0.1	0.2	0.3	0.3	0.2	0.2	0.2	0.1	0.1	0.1	0.0

We use this function to determine the optimal response of player $n-1$, as shown below:

g_{n-2}	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y_{n-1}(g_{n-2})$	0.4	0.3	0.2	0.4	0.3	0.2	0.1	0.1	0.2	0.1	0.0

As an example, suppose $g_{n-2} = 0.3$, and $y_{n-1} = 0.4$. Then $g_{n-1} = 0.3 + 0.4 = 0.7$, and the last player (player n) chooses $y_n = 0.1$. The profits of the penultimate player (player $n-1$) are then $0.4/(0.3+0.4+0.1)-0.4 = 0.1$. If instead player $n-1$ chose $y_{n-1} = 0.3$, then $g_{n-1} = 0.3+0.3 = 0.6$, $y_n = 0.2$ and the profits of player $n-1$ are $0.3/(0.3 + 0.3 + 0.2) - 0.3 = 0.075$. Note that the function $y_{n-1}(g_{n-2})$ gives the optimal response of player $n-1$ for any g_{n-2}, regardless of the number of players. This independence permits recursive search for each player's optimal payment.

Partly because the function $y_{n-1}(g_{n-2})$ has several local maxima rather than a unique one, we could not find a general analytic solution. But even if such a solution exists, we must beware that firms contribute not equations, but definite dollar amounts. If the numerical solution to an equation varies with the algorithm used or with the number of significant digits computed, then we would want to consider how the solutions vary with the computational costs incurred.[1] The qualitative results we obtain below may then also apply when firms use explicit equations to determine their contributions.

For our numerical solutions we consider a prize of 1 and a grid of 50,000. The following table shows optimal payments by player i when n players participate. The column headings show the number of players. The entry in a row labeled i is the payment by player i, rounded to three significant digits.

One empirical test of the Stackelberg model rests on examining the pattern of payments. The table shows that for $n > 2$ the first mover spends more than later movers, and that the first two movers spend more than later movers. The available data are consistent with this result. Thus, in the 1985/86 electoral cycle, average early contributions by corporate PACs are $476, while the average for late contributions is $271. Similarly, early contributions by labor union PACs average $238, while the late ones average less than half, $93 (McCarty and Rothenberg, 1995).

Stackelberg payments
Number of players

i\n	2	3	4	5	6	7	8	9
1	0.250	0.365	0.386	0.400	0.476	0.371	0.947	0.945
2	0.250	0.260	0.258	0.252	0.203	0.222	0.003	0.001
3		0.165	0.163	0.157	0.144	0.176	0.005	0.003
4			0.091	0.091	0.085	0.111	0.007	0.005
5				0.049	0.044	0.059	0.010	0.007
6					0.023	0.030	0.011	0.010
7						0.016	0.008	0.011
8							0.004	0.008
9								0.005
Total	0.500	0.791	0.899	0.949	0.976	0.984	0.996	0.996

Stackelberg profits
Number of players

i\n	2	3	4	5	6	7	8	9
1	0.250	0.096	0.044	0.023	0.011	0.006	0.004	0.004
2	0.250	0.069	0.029	0.014	0.005	0.004	0.000	0.000
3		0.044	0.018	0.008	0.003	0.003	0.000	0.000
4			0.010	0.005	0.002	0.002	0.000	0.000
5				0.003	0.001	0.001	0.000	0.000
6					0.001	0.000	0.000	0.000
7						0.000	0.000	0.000
8							0.000	0.000
9								0.004

5. Will firms play a Stackelberg game?

Pursuing the last point, we can inquire whether any firm will want to move first, or more generally to move early. An early payment is advantageous if the following conditions are satisfied: (1) the first mover earns more than later movers; (2) the first mover earns more than in a Nash equilibrium; (3) a firm with the opportunity to move second prefers to move second than to move later (and so on); (4) the advantages of moving first are robust in the sense that the first mover does better than later movers even if later movers do not move sequentially.

We see from the table that the first condition is satisfied: the first mover's profits are at least as large as the profits of later movers. A firm given the opportunity to move first (perhaps because of slight differences between the firms, or because of random events) will prefer to move first rather than later.

The second condition, involving a comparison of profits in a Stackelberg solution to profits in a Nash solution, need not be satisfied: profits of the first mover are never higher than in a Nash equilibrium. For $n = 2$ the Stackelberg solution is identical to the Nash equilibrium. For $n = 3$, the first mover's profits are about 13 percent smaller than a firm's profits in a Nash equilibrium. For $n \geq 6$ the first mover's profits are less than half a firm's profits in a Nash equilibrium. The profits of later movers are even smaller.

The third condition, which considers the incentives of later movers, is not always satisfied. That is, if one firm does move first, succeeding firms may not want to move sequentially. Computations show, for example, that in a sequential game with 5 players and a grid of size 1,000, the second mover's profits are 0.0016, which is less than the third mover's profits (0.0020) and less than the fourth mover's profits (0.0017). So in this example no firm would want to move second.

A firm that did not move first can effectively avoid moving second by hiding its payments or by delaying until the last moment the payments it makes. The model given by Dixit (1987), and discussed further below, may then be more general than he indicated. We need not *assume* that one firm is the leader and that the rest then play a Nash game. Instead, such a Stackelberg-Nash game can result from the incentives of firms.

To examine the fourth, related, condition, which asks whether the benefits of moving first are robust, we consider a game in which one firm moves first, and the remaining firms play a Nash game, given the payment made by the first mover. Such a game was analyzed by Dixit (1987), who finds that for $n > 2$ the first mover's profits are higher than a firm's profits in a Nash equilibrium. To analyze the Dixit game further, consider for $i = 2, ..., n$ firm i's optimal strategy given aggregate payments $Y - y_i$ by the other firms. As with a Nash equilibrium, the first-order conditions for a maximum satisfy (1), namely,

$$y_i = Y(1 - Y) \quad i = 2, ..., n. \tag{2}$$

Sum over $i = 2, ..., n$ to get $Y - y_1 = (n-1)Y(1-Y)$ or

$$y_1 = Y(1 - (n-1)(1-Y)) . \tag{3}$$

Differentiating with respect to y_1 yields

$$1 = \frac{dY}{dy_1}(1 - (n-1)(1-Y)) + Y(n-1)\frac{dY}{dy_1}$$

or

$$\frac{dY}{dy_1} = \frac{1}{2 - n + 2(n-1)Y} . \tag{4}$$

We now turn to Firm 1's strategy. It aims to to maximize $\frac{y_1}{Y} - y_1$ where Y and y_1 satisfy (3) and (4). Substituting (3) we find that Firm 1's objective is to maximize $1-(n-1)(1-Y)-y_1$ or equivalently, to maximize $(n-1)Y-y_1$. Differentiating with respect to y_1 yields the first-order condition

$$\frac{dY}{dy_1} = \frac{1}{n-1} .$$

Substituting (4) gives $1/(2 - n + 2(n-1)Y) = 1/(n-1)$ or

$$Y = \frac{2n-3}{2n-2} .$$

Making use of (3) and (2) yields

$$y_1 = \frac{2n-3}{4(n-1)}$$

and

$$y_i = \frac{2n-3}{4(n-1)^2} \quad i = 2, ..., n .$$

From the viewpoint of the first firm, the solution is surprisingly simple. Note that $y_1 = \sum_{i=2}^{n} y_i$, or that firm 1 makes half of all rent-seeking expenditures. Thus, the behavior of the other firms aggregates to the same behavior of a firm in a Nash game with two firms.

Aggregate profits of the firms are

$$1 - Y = \frac{1}{2(n-1)}$$

as opposed to $1/n$ under the Nash game.

Numerical results are shown below. For the complete sequential game we use a grid of 50,000. The Nash and Dixit games can be solved analytically, and so evaluated with any degree of precision; we chose to calculate the values with ten significant digits.

Profits of first mover under Sequential, Nash, and Dixit games
Number of players

	2	3	4	5	6	7	8	9
Sequential	0.250	0.097	0.044	0.022	0.011	0.006	0.004	0.004
Nash	0.250	0.111	0.062	0.040	0.028	0.020	0.016	0.012
Dixit	0.250	0.125	0.069	0.041	0.050	0.041	0.036	0.023

We see that profits in a Nash equilibrium are always greater than in a complete sequential game, and that the first mover's profits are higher in a Dixit (partial sequential) game than in a Nash equilibrium. Thus, if a first mover could be assured that successor firms would play Nash, the first mover benefits from moving first. But if it believes that later firms will also behave sequentially, the first mover would have a disincentive to move first – it would prefer to play a Nash game.

Of course, the sequence of moves need not always be under a firm's control. Large, bureaucratic, firms may necessarily take longer to execute a decision, and may be more prone to leaks about their plans. A large firm which aims to make a contribution before the election may therefore find its plans known to others quite early. It would effectively be an early mover, even if it did not want to be. The amount it contributes, however, could be under its control, and it should consider the effects we analyzed.

6. Conclusion

One contribution of this paper is to the positive analysis of rent seeking. We find that all firms may make higher profits in a Nash game than in a Stackelberg game. The numerical solutions also suggest that earlier movers make higher profits then later movers. Since we have determined numerical solutions for different grid sizes, we can also ask how those profits vary with computational effort, where greater computational effort can be reasonably interpreted as use of a finer grid. We find no regular pattern. In some ranges profits increase with the size of the grid, while in other ranges they decrease. For example, the profits of firms decrease when the grid size increases from 500 to 508, but profits increase when the grid is further increased to 509. Even large grids, which we can interpret as weak bounded rationality, can lead to outcomes that significantly differ from those that obtain with much larger grids. Finally, within some ranges, the profits of all firms may decline as they refine their computations.

Note

1. For another example of how the units of calculation can matter, see van Damme, Selten, and Winter (1990). They show that that in Rubinstein bargaining game with a sufficiently low discount rate, *any* partition can be a subgame perfect equilibrium.

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