GRAPH THEORY

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Fall Semester 2012

Course number: 0366.3267.
When and where: Wednesdays 15-18, Dan David 204.
Prospective audience: the course is intended for second and third year undergraduate students in Mathematics or Computer Science.
Prerequisites: first year courses in mathematics, most notably Discrete Mathematics or Introduction to Combinatorics.
Requirements and grade: Homeworks will be given once every two-three weeks; submitting at least three quarters of them is mandatory; the final grade will be composed from the final exam's grade (90%) and homeworks (10%).

Syllabus

1. Basic notions: graph, graph isomorphism, adjacency and incidence matrices, paths, cycles, connectivity, subgraphs. Vertex degrees, Sperner's lemma and Brouwer's fixed point theorem.
2. Trees. Equivalent definitions of a tree. Spanning trees. Cayley's formula, matrix-tree theorem.
3. Connectivity. Vertex connectivity and edge connectivity. Degrees and connectivity, Mader's theorem. Blocks, 2-connected graphs. Menger's theorem.
4. Eulerian and Hamiltonian graphs. Theorems of Dirac, Ore, Chvátal-Erdős.
5. Matchings. Theorems of Hall and König for bipartite graphs. Tutte's theorem.
6. Colorings. Vertex colorings, degrees and chromatic number. Greedy algorithm. Brooks' theorem. Color-critical graphs. Graphs with high girth and high chromatic number. Edge coloring, theorems of König and Vizing.
7. Ramsey theory. Ramsey's theorem. Upper and lower bounds for Ramsey numbers. Examples of Ramsey-type theorems.
8. Extremal graph theory. Turán's theorem.
9. Planar graphs. Euler's formula. Kuratowski's theorem. Coloring planar graphs.

Bibliography

1. J. A. Bondy and U. S. R. Murty, Graph theory, Springer, 2008.
2. R. Diestel, Graph theory, Springer, 2010, or earlier editions.
3. L. Lovász, Combinatorial problems and exercises, 2^{nd} edition, North-Holland, 1993.
4. D. West, Introduction to graph theory, 2^{nd} edition, Prentice Hall 2001.