

## Homework 1: Nov 12, 2011

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**Homework number 1.**

**Question I:** Assume that  $R(x)$  is strictly convex and let  $f_\tau \in R^N$  be arbitrary vectors. Assume that  $y_t$  has the property that  $\nabla R(y_t) = -\eta \sum_{\tau=1}^t f_\tau$ . Show that,

$$\arg \min_{x \in K} \left[ \sum_{\tau=1}^t f_\tau \cdot x + \frac{1}{\eta} R(x) \right] = \arg \min_{x \in K} B^R(x \| y_t)$$

**Question II:** Compute a subgradient of  $f(x) = \max_{i=1, \dots, m} |a_i^T x + b_i|$ , at any point  $x$ . (Sufficient to find one subgradient, no need to characterize all of them.)

**Question III:** Show that in Follow The Leader (FTL) the regret is bounded by the number of changes in the *best action* (assuming that the losses are in  $[0, 1]$ ).

**Question IV:** Consider Follow The Perturbed Leader (FTPL) for a quadratic optimization function, i.e.,  $f_t(x, w_t) = \sum_{i=1}^N \sum_{j=1}^N w_{i,j,t} x_i x_j$ .

Show how to use the linear FTPL for this setting.

What is the regret bound?

(You can use the bound shown in class of  $\Omega(\sqrt{RADT})$ , where  $D \geq \|d_1 - d_2\|_1$ ,  $R \geq |d \cdot s|$ , and  $A \geq \|s\|_1$ .)

(Hint: map the problem to a higher dimension.)

**The homework is due in two weeks**

**Research Question**

The question here are intriguing research questions (not part of the regular homework)

**Challenge 1:** There must be a simpler way to do the approximation algorithms!

**Challenge 2:** Try to derive an extension to FTPL for non-linear functions.