Assignment 2 - Computational Geometry (0368-4211)

Due: May 4, 2015

Problem 1

Modify the randomized incremental algorithm so that it computes the convex hull of a set of n points in $\mathbb{R}^4$. Explain how the modified algorithm works, and show that the expected number of facets (3-dimensional faces) that it generates is $O(n^2)$. (No need to go into details of the data structure (the suitable generalization of the DCEL structure) that represents the convex hull, and its maintenance as points are inserted.)

Problem 2

The *normal diagram* of a 3-D convex polytope K with n vertices is defined as a partition of the unit sphere $\mathbb{S}^2$ in $\mathbb{R}^3$ into regions $R(v_1), \ldots, R(v_n)$, one for each vertex v_i of K, so that a direction $u \in \mathbb{S}^2$ is in $R(v_i)$ if the plane supporting K and having u as its outward drawn normal touches K at v_i.

(a) What are the edges and vertices of this partition? How many are there? How fast can the diagram be computed from the DCEL representation of K?

(b) The *width* of K is defined as the smallest distance between a pair of parallel supporting planes of K. Prove that the width is always attained by two planes that satisfy one of the two conditions: (i) One plane passes through a face f and the other passes through a vertex v of K, or (ii) One plane passes through an edge e and the other passes through another edge e' of K.

(c) Give an example of a polytope K for which the number of pairs (e, e') of edges that have parallel supporting planes (one plane touching e and the other touching e') is $\Omega(n^2)$.

(d) Use the normal diagram and the sweeping technique to compute all candidate pairs of planes that satisfy (i) or (ii) in (b), and thus to compute the width of K. How fast is the algorithm? (**Hint:** Think of superimposing the normal diagram with a copy of itself.)

Problem 3

(a) Use line sweeping to solve the following problem: Given a set S of n points in the plane, and a radius R, find a disk of radius R that contains the maximum number of points of S. What is the running time of the algorithm? (**Hint:** Use a dual representation, where the roles of disks and points are interchanged.)

(b) Using (a), solve the "converse" problem: Given S as above and a parameter $k \leq n$, find a disk of smallest radius that contains k points of S. (**Hint:** Use binary search.)

Problem 4

Let P be a simple rectilinear polygon in the plane, with a total of n edges. That is, P is a closed polygonal curve that does not cross itself, and every edge of P is either horizontal (parallel to the x-axis) or vertical (parallel to the y-axis). Let Q be an arbitrary set of n pairwise disjoint segments in the plane. Count the number of intersections between P and Q in $O(n \log n)$ time, using line sweeping. (Note that the actual number of intersections can be quadratic in n.)