COMPUTATIONAL GEOMETRY - FINAL EXAM

Prof. Micha Sharir
School of Computer Science
Raymond and Beverly Sackler Faculty of Exact Sciences
June 5–8, 2003

Answer four of the following problems. All problems have equal weight (25 percent). Return the exam to Ruti Friedberg in the Mazkirut Studentim no later than Sunday, June 8, 12:00. Good luck!!

Problem 1

Let S be a set of n pairwise disjoint segments in the plane. Describe a sweep-based algorithm that finds a vertical segment of length 1 that intersects the largest number of segments in S. (An algorithm that runs in near-quadratic time is acceptable.)

Problem 2

(a) Let S be a set of n points in the plane. Preprocess S so that, given a (non-vertical) line ℓ, one can determine whether there is any point of S above ℓ in time $O(\log n)$.

(b) Same problem, except that now the query is a line segment e, and we want to determine whether there is a point of S above e, in $O(\log^2 n)$ time. (A point w lies above a segment e if w lies above the line containing e and its x-coordinate lies between the x-coordinates of the endpoints of e.)

(**Hints:** For (a), use duality. For (b), try to decompose a query into $O(\log n)$ independent 'sub-queries', based on the x-coordinates of the points, where each sub-query can assume that e is a full line; then apply (a).)

Problem 3

Let K be a given convex polyhedron in 3-space having n vertices, and let e be a given segment. Let K_e be the region swept by K when it is translated by e. Formally, it is the set

$$\{x + y \mid x \in K, y \in e\}.$$

Describe the geometric structure of K_e and give a linear-time algorithm for computing it (say, from a DCEL representation of K construct the DCEL representation of K_e).

Problem 4

Let S be a set of n points in 3-dimensional space, let C be its convex hull, and let w be a specific vertex of C that lies on its upper portion. We are also given a vertical line ℓ in space. We want to find the lowest point v on ℓ from which w is visible (that is, the line segment vw does not meet the interior of C). Show how to reduce this problem to a linear programming problem, and thus solve it in $O(n)$ time. (Note: C is not given, only S is given!)

(**Hint:** Think in terms of finding a plane supporting C at w.)

Problem 5

Let S be a set of n axis-parallel rectangles in the plane, so that the bottom edge of each rectangle in S lies on the x-axis.

(a) What is (an upper bound on) the combinatorial complexity of the union K of the rectangles in S?

(b) Give an efficient algorithm for computing the union and its area.

Problem 6

Let D be a set of n pairwise disjoint disks in the plane. Describe how to adapt Fortune's line sweeping algorithm to compute the Voronoi diagram of D in $O(n \log n)$ time. (Go over the various steps of the algorithm, and explain how to modify them for the case where the sites are disks. Explain also the analysis of the algorithm in this case.)