Assignment 2 - Geometric Optimization (0368-4144)

Due: December 3, 2013

Problem 1

Let A and B be two sets of n points each, where the points of A lie on the x-axis and the points of B lie on the y-axis. Let k be an integer between 1 and n^2. The goal is to find the k-th smallest distance between a point in A and a point in B.

Solve the problem in near linear time using three approaches: (1) parametric searching (plus Cole's improvement); (2) monotone matrix searching; (3) a randomized approach. Explain each solution.

Problem 2

Apply the algorithm for searching in totally monotone matrices to solve the following problem in $O(n \log n)$ time: We are given a set P of n points lying on a closed convex curve C, and the goal is to find the triangle of largest area whose vertices are three of the points of P. To simplify the solution, consider only triangles abu, where a and b lie on the lower portion C^- of C and u lies on the upper portion C^+.

The solution is not simple. Here are some guiding steps:

(a) Show that, for a given base ab, the third vertex u which maximizes the area of $\triangle abu$ can be found in $O(\log n)$ time (possibly after some (cheap) preprocessing).

(b) For each base ab, with $a, b \in P \cap C^-$, let A_{ab} denote the maximum area of a triangle abu, with $u \in P \cap C^+$. Show that A has the inverse Monge property.

(c) For (b), fix a vertex u in $P \cap C^+$, and four vertices a, b, c, d in $P \cap C^-$, in this left-to-right order, and prove that

$$\text{Area}(adu) + \text{Area}(bcu) \leq \text{Area}(acu) + \text{Area}(bdu).$$

Complete the details of implementing the algorithm.

Problem 3

Apply Chan's technique to solve efficiently (in close to quadratic time) the following problem. Let A, B, and C be three sets of points in the plane, each of size n. Find the triangle uvw of smallest area, such that $u \in A$, $v \in B$, and $w \in C$. (**Hint:** For the decision procedure, try to use duality.)

Problem 4

Let P be a set of n points in the plane, all lying inside the square $S = [-1, 1]^2$. Give an efficient (near-linear) algorithm that finds an axis-parallel rectangle R of largest area that is (i) fully contained in the interior of S (and does not touch its boundary), (ii) contains the origin, and (iii) does not contain any point of P in its interior.

Note: This extends the simpler case shown in class. Classify the possible solutions according to the quadrants that contain the four points of P that the boundary of such a rectangle touches. One of these cases has been treated in class, but here too some care is needed, because of the points in the other two quadrants.