I am 80 To-Day.

Vitali Milman.

After the last classes, 2007

Contents

1.   Introduction.

Ten years ago I wrote an essay called TO-DAY I AM 70. Back then I was seriously interested in answering the question "who am I?". Our Ego is often a noticeable obstacle blocking the integrity of the whole of the picture. And, although by my 70 my Ego had shrunken quite a bit, the very question being posed meant that it was, still, a blockage to my ability to clearly see the picture.

Today I am no longer interested in knowing who I am. I am aware of the answer and I accept it.

In Appendix 2, I will bring some opinions about my research and the areas of Mathematics I, actually, opened: "Concentration Phenomenon" and "Asymptotic Geometric Analysis". Most of these opinions are anonymous, and I explained there, why they are typical.

But it may be a curios experience, glancing at the 10 years that I have just left behind. Yet, I will start from the back end, from this summer. At the end of July, celebrating my 80th anniversary, my students had arranged a conference in my honor; it was held from July 29 to August 2. It was, really, Shiri Artstein (-Avidan), who took upon herself the whole load of arrangements. In 2004, under my supervision, she defended her Ph.D. thesis, and today she is the youngest full professor at our University, maybe, even, in the whole Israel. She is quite a remarkable young mathematician, who, along with growing her three children, has already a lot of discoveries of her own, and has a few of her own students. She is a recipient of the Erdos Prize for the young Israeli mathematicians, and a few years ago in one of the anonymous reviews related to her grant application I had chance to read that she is likely to become the first woman with Fields medal. Although this has not happened, her level is coherent with these expectations.

Shiri Artstein-Avidan, 2019

The members of the scientific and organizing committees for the above mentioned conference were my students, and they made an interesting decision to mostly form the conference based on presentations by the scientists, who defended their Ph.D. thesis under my supervision, or, who started their scientific careers under my influence (there had been collected nearly 20 of such presentations). The rest made a group of very close to me super-experts in different branches of science. The conference was remarkable and included presentations related to medical research (I too had publications in it), philosophy (one of my students turned out to be a philosopher), protein structure and reconstruction (again, one of my students), etc.

A very nice and special event at the Conference was the piano concert given by Ingrid Carbone. She is a mathematician by profession and an excellent pianist, and she decided to become a professional musician too. She very successfully succeeded in her decision! Her concert (she played Franz Liszt) was excellent: it was a remarkable performance, and we all thoroughly enjoyed it. We just loved it (and her!)
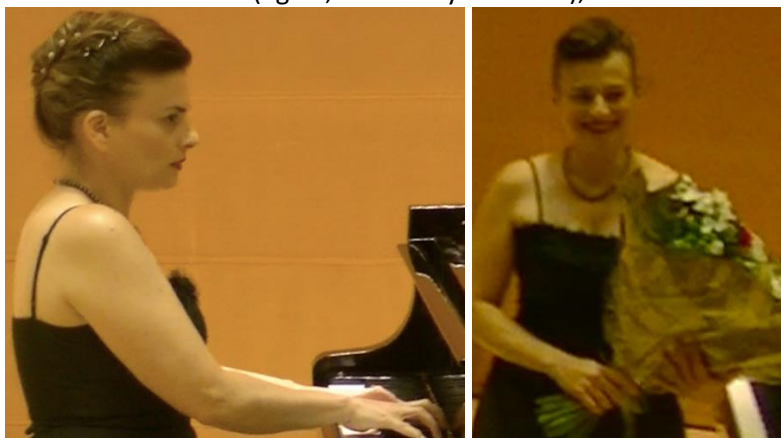
Ingrid Carbone, 2019

In the last 10 years, from 2010 to 2019, I had 6 new PhD students (Sasha Sodin, Ronen Eldan, Alex Segal, Dmitry Faifman, Dan Florentin and Liran Rotem), who are today professors in Israel, Canada, England and USA. I can hardly believe this myself that I could do it. Well, I felt in some cases that for better development of these young mathematicians I had to invite second supervisors with their tastes in Mathematics more corresponding to that of these young scientists. And my former students took the roles of such supervisors upon themselves: Boaz Klartag joined his efforts with mine to supervising Ronen, Shiri Artstein helped me with supervision of Dan, and Semyon Alesker – with supervision of Faifman. I think all sides gained from these connections, were happy with them, and each of the mentioned joined supervisors has very high quality research papers published with their mentioned students.

I should say that I am extremely proud with the family of my students. I could not imagine in my dreams that I could produce such distinguished group of researchers. The formal list may be seen on the webpage:

Of course, two people on this list arrived to me already ready mathematicians (I mean, Leonid Polterovich and Alexander Reznikov) and I was, mostly, needed for formality of signing their Theses, but, also, (already not formally) to take care of them at the start of their career. On the other hand, there are two other very top mathematicians (Noga Alon and Apostolos Giannopoulos), who are not on my formal list, but who, de-facto, were under my advices in their first steps in Mathematics.

Every presenter at the conference, of course, had a few warm dedicated to me words. This is a tradition. But some found a way to go beyond tradition. I will mention just two such cases that look to me more original, than others. First is by Noga Alon who in his talk "plays" a meeting of our School long ago to demonstrate the level of my involvement and abilities. It may be seen in the video of the conference where his talk is the first (N.Alon, "Vitali, traces and geometry"). The video may be seen at

https://www.youtube.com/playlist?list=PLNiWLB_wsOg6DCKedeXjvNP-6UKSGjWT3&disable_polymer=true

Also all other talks that were given in Tel Aviv may be seen there. The remaining were by the Dead Sea.

The second case I mean belongs to Ronen Eldan, who compared having me as a supervisor to carrying the joker card through one's career: "as all of us know, arriving to an unfamiliar territory for the scientific research presentation feels quite uncomfortable – strangers and strange circumstances are all around; but the moment you name Milman as your supervisor everything changes as if by the magic wand, the new place turns into your home and everyone wants to come and tell you something nice" (from my recollection in my own words, but this carries the true essence of what was said).

Many of my former official and not official students (I had many of these, who achieved extraordinary high levels in their careers) mentioned on social networking forums my given to them useful advices. Some requested that I write to them with such advices, and on some occasions I did. I will try to collect them in the Appendix 1 after the main text.

But I would like to end this Introduction with the fable that I call

The story of a very old Oak.

Many times I walked through the forests, and, more, than once, came across a meadow with a lot of young oaks. Nice, I thought each time, beautiful. Yet, each time I'd immediately forget about the meadow. Nothing outstanding was there for my eyes to remember, to fix my attention. Not a single young oak of this meadow I can recall.

But when I come across an old oak, its beauty, strength and power stay in me for years. One oak near Hevron, the Oak of Abraham, I remember in all its details even now, 42 years after that frozen moment, when I stayed near it and could not leave. This huge oak was active and alive, and it was, I presume, much older, than thousand years old. But it was grown from another oak, from its bark, huge piece itself. It was connected to even larger and looking much older trunk, which lied on the ground. Although it is hard for us to imagine, yet, that trunk was probably many thousands years old, and Abraham, indeed, had set his tent in the shadow of the oak there, as the story says.

The majestic oak radiated strength and power. THIS was the beauty to remember, to carry through your life.

The comparison between young and old oaks comes to my mind, when I think of our very old age, and look around on young generation. The beauty of accumulated passing time is with us!

2.   80 passing years!

I reread now the "Looking Back on 80 years" essay by Somerset Maugham. And I decided to follow him in reflecting on what strikes me as looking back at my 80 years I lived through. Similarly to Maugham, I am stricken by how had changed our world through those years. Maugham entered this world in 1874, and left it in 1965. A lot of new technologies were discovered and entered people's life during that period, and life changed drastically.

What could humanity expect coming next, after 1965?

We already had phones (not, yet, well spread in the Soviet Union, where I lived), TV, people already went to Cosmos with a big hope for its development. We had computers (they were of the size of 3 floor building back then), which could compute some, otherwise unsolvable, problems.

In that year, in 1965, I received my PhD. Under influence of fiction movies, we could dream of wireless phones, and even video-phones. However, honestly, I never believed that this could be, really, done, and that

I would see such things during my life-time. (Of course, I have such phone now in my pocket). Life had overpassed our dreams. Many things we never thought were technically possible became a reality. And the speed of development only increases. We, the older generation, are completely lost in this novel World. Things are changing too quickly for us to have time to get used to these changes, to adapt ourselves. And my 8-years old grandchild helps me to change TV mode from, say, computer version to standard TV show. And my two years old grandson already knows, how to open a smartphone to see pictures or his movie. But I cannot do it for him. Our 40-years old children laugh, when we ask them to fix our tablet or smartphone, when we pushed some wrong button and don't know, how to return things back. And at such moments I am curious about what new progress, new discoveries await for in the future that will make these today small grandchildren laugh at my son, when he will have a similar request to them 40 years from now?

Young generation, and I mean now age 30+, cannot imagine how we lived around 30 years ago. Once I told to a reporter (science reporter!) of NY Times a story from around 1985 about someone (who he asked me about) that he took a piece of paper and put down in writing on it some computations. In his article this reporter wrote that the person took his laptop and quickly computed... . Sorry! We did not, yet, have laptops at that time! We used papers to write and compute. This reporter could not imagine such time.

Actually, in Mathematics I observed a sudden explosion of discoveries around 1990, and it appears to me that this period is not, yet, behind us. I was on the Panel of a section "Operator Spaces and Functional Analysis" for the International Congress in Zurich in 1990. Preparation for it was going around 1988/1989. Usually, it is a very non-trivial job to select speakers between a large group of excellent researchers and there happen serious "fights". However, there is an easy part: whoever solved a well-known very longstanding problem must present it at the Congress. Usually, such cases would compose only a small part of all the open spaces in the list. But that time around, such cases filled the whole list! I checked with a few more Panels and heard the same. And indeed, the Last Fermat Theorem (Wiles) and Poincare Conjecture (Perelman) were solved shortly later. The progress and discoveries were everywhere. Something is going on and I am sending you to Chapter 6 for the discussion on the matter.

Of course, email connection brought us much more close to each other, and joint research becomes typical. We slowly become a part of a "super brain" of a large group of researchers working on specific problems. Almost all research is now done in discussions of groups, and, sometimes, very large groups. This creates much higher brain power in working on a single problem. But this itself does not, yet, explain, what we observe in recent years.

Also, the use of technology entered our life in Mathematics. In old days I presented my talks just with a chock near a blackboard. I used in advance prepared slides the first time only on my first Congress talk in Berkeley in 1986. But I think, the second time it was already 10 years later on my Plenary talk in Budapest at the European Congress in 1996, and then, again, two years later in 1998 at the Berlin Congress. It seems to me that from that time on I used slides regularly. However, I think that until 2010, or even slightly later, I did not use the computer presentation. Now I cannot imagine how one may not use computer presentations for his/her talks. Of course, I am doing it still very primitively, without motion pictures, photos or other tricks.

Of course, our everyday life changed drastically. One would unlikely run across a walking person without a phone near his/her ears. And my game now is to estimate the IQ of a person by the time s/he speaks by phone walking. (I think this follows the reverse proportion rule: the more a person is on a phone walking, the less is his/her IQ). This is already starting to change. The new fashion is to type and read messages on smartphones. Before people did not hear anything, as they walked. Now they don't see anything, since they look all the time down at their smartphones, as reading/typing messages. Kids are walking and playing games on their smartphones. This is very dangerous, but I don't see that anything can stop it. How many of them will grow up to be addicted to games?

We already see cars on the streets without drivers, and once, two years ago, I saw two small robots walking one after another on the street of Berkeley. People stopped for a second to look, but were not too surprised. I walked after these robots for a block. They accurately avoided obstacles on their way, and continued to do their job. I could not locate anyone who would control them on the street. This was a bizarre picture from the fiction movies. Obviously, we will soon see cars that fly. They, actually, do, already exist, but are, still, too expensive, and regulations are not, yet, written on how to use them publically. So, we may predict some development for 5, perhaps even 10, years ahead. But, surely, not beyond that.

I think, our society is more and more divided into people of creative professions (scientists, people producing culture), businessmen and people, who just care to have good salary ("parnasa" in Hebrew; I hate this word). Of course, in Israel a very important stratum is military, and in every society there will be medical personal. But, also, them, as well as people of other professions, say, lawyers, divided into the above

categories in terms of their spirit and the way of thinking and seeing life. Yes, politicians appear to be a special group, of which I don't want to say a single word.

It is also interesting to observe how people dress. Mathematicians these days dress not formally. Young generation's fashion is to give talks dressed "Bohemian": intentional holes in pants, shirts that look dirty, although they are not, etc. This brings to my memory an episode in my life. I gave many talks at Pisier seminar in Paris 6/7. I always considered that (my) talks should be given in a more formal manner, including that I would always be well-dressed. Once Pisier asked me: "Vitali, why do you always give your talks especially well dressed, almost formally" (for instance, I'd always put on my best available suit with jacket). My response was: "I want to show to young generation, Gilles, that one may prove good theorems being well dressed." Gilles laughed.

Gilles Pisier, around 1984

Returning to our drastically and dramatically changing life, I would say, that YES, I would like to live a little bit more to see, where all this is going. More comments on this see at the end of Chapter 6.

3. "Aging" Conferences with some of my memories of friends and colleagues, and, especially, of Jean Bourgain.

During my mid age I liked to participate in many conferences, and, in general, to give talks on conferences, seminars and colloquiums. Every year during 3 or 4 decades I standardly gave 10-15 lectures abroad. Of course, it changed recently and I joke now that before I gave talks to be (well) known, but in recent years I gave them not to be forgotten.

However, the reasons for talks dramatically changed recently. Now most conferences I attend are in memory of friends and colleagues. Some of them, and that's luck!, are not in memory, but in honor of someone's 70-th/80-th or even 90-th or more anniversary.

The mentioned in memory of conferences are emotionally very charged, especially, if they are less, than a year or two after the tragic event. The most recent and also the most difficult for me conference was in the memory of Jean Bourgain. I will write later about it and about him.

There was, also, an "easier" conference in June devoted to the 100 years anniversary of Pogorelov. It was "easier", as he passed away already long ago.

The conference was in Kharkov, the place he worked after the Second World War, and until his retirement and death. He carried a very interesting personality, and was a special person for me. Pogorelov was the Chairman of the Admissions Committee for mathematical students in Kharkov University in 1956, when I entered this University. This was the time of the so called "Khrushchev's spring". But the situation was not, yet, clear regarding Jews, and it was not obvious that I would be admitted to university; this was despite that I won first places on many high school Olympiads (competitions) on Math and Physics. But Pogorelov personally took over my exam, and I received 5 + 5 (maximal grades for written and oral exams), and, consequently, became a student. I have many nice words to say about Pogorelov. I, really, liked this person, and I am so glad that I had an opportunity to invite him to us, to Israel, in 1996 during a very difficult time in Ukraine. My talk on Pogorelov's conference opened the scientific part of the conference, and I feel proud about this.
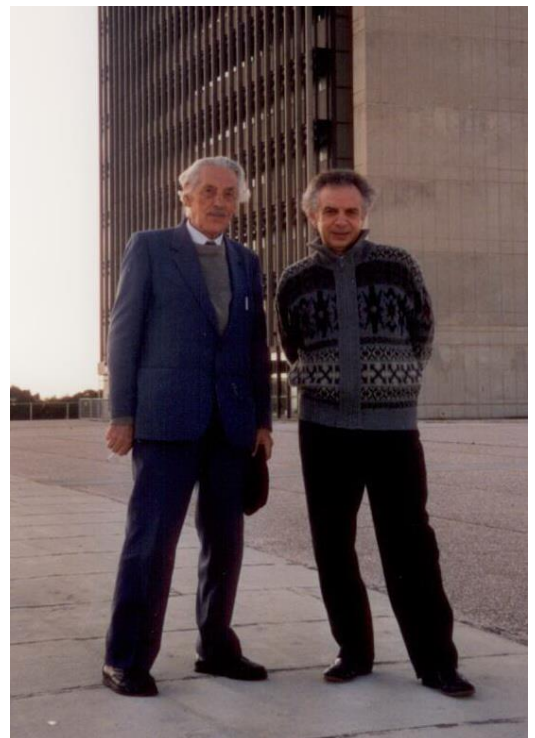
A.V. Pogorelov and Vitali Milman, 1996, Haifa

My talks, actually, had opened most of the conferences I participated in for the last 30 + years, including the memorial conferences for J.Lindenstrauss, A.Pelszinski , V.Havin and others.

There is a tradition of AMS to collect, when some well-known member dies, some short stories-memories from the friends and co-authors, and publish them in Notice in one article. I was asked to write them for a few people. Below I bring in three such memorial stories: on Joram Lindenstrauss, A. (Olek) Pelczynski and Jean Bourgain (in the chronological order of them passing away). I will also add many more memories and "stories" on Jean.
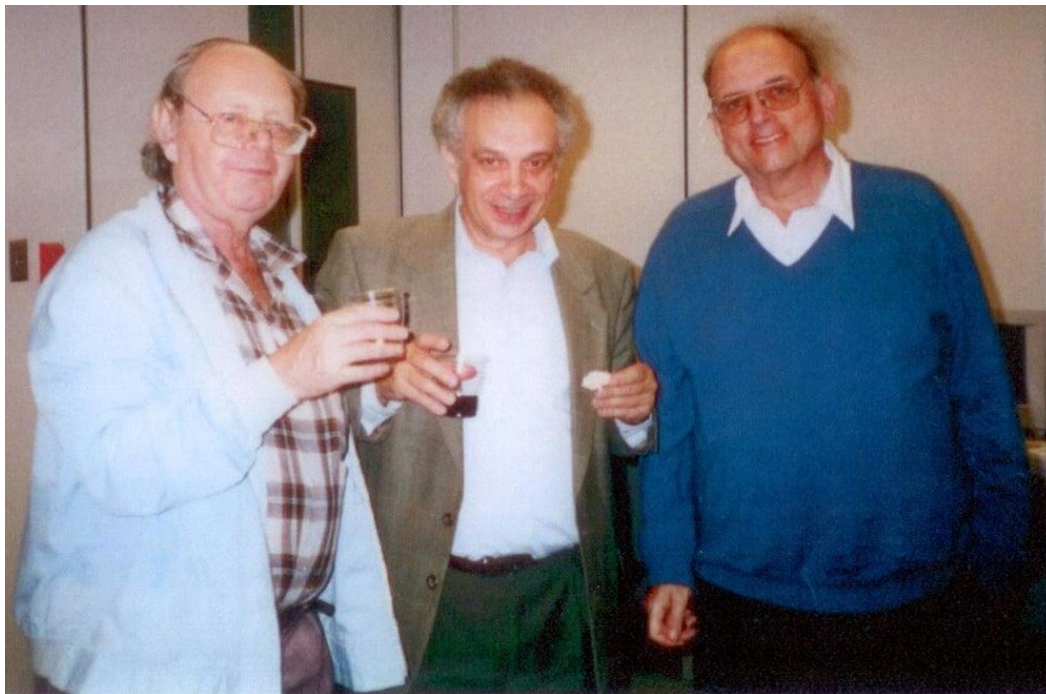
Olek Pelczynski, Vitali Milman, Joram Lindenstrauss, Summer 1999, Vancouver, my 60th

Joram Lindenstrauss (October 28,1936 - April 29, 2012)

It was the ICM-1966 in Moscow. A lot of mathematicians arrived from the West, but my highest expectation was to meet Dvoretzky and Lindenstrauss. I knew well one of the first papers by Joram about duality for the moduli of convexity and smoothness, and I, also, read all of his work that I could find in our (poor) libraries. However, Dvoretzky, indeed, arrived, but Lindenstrauss did not. Dvoretzky told me that "they" (Russian authorities) wrote to Joram that there is no room in hotels (?!) and they cannot let him in.

So, the first time I met Joram was in Israel, in 1973 after my emigration. It was a very difficult time after the Yom Kipur War. My family stayed in a dormitory for new immigrants in Tel Aviv. Once someone knocked on our door. I opened and saw a young extremely nicely looking person, who looked at me and said "Joram Lindenstrauss". I remember this moment well after ~ 40 years. I lost my voice, and I hardly remember the continuation of our first meeting. Despite hundreds of days we spent together later, and despite the passing of 40 years, that first image of Joram stays in my mind, comes to my mind, when he is mentioned, and it is not shadowed by later naturally coming changes.

Our serious scientific cooperation started 2 years later (I needed this time so that to learn Hebrew and English, at least, up to the level, at which I could understand a little bit of both), and resulted in joint papers with Figiel (Figiel – Lindenstrauss - Milman) in 1976 (AMS Bulletin) and 1977 (Acta Math). I heard opinions that these were the most significant results in Geometric Functional Analysis in the 1970s. I had learned a lot from working on this paper with Joram, I was learning from his broad knowledge and his taste. I felt that I became a different mathematician at the end of this period. Unfortunately, the cooperation stopped and returned only 10 years later. We, actually, prepared some directions and ideas for working together, and I even wrote a few pages of notes. But one young mathematician heard the discussion on these results from Joram, quickly wrote a paper on them and submitted it. Joram was very angry, but did not want to intervene (I would, with my character) and the "corner-stone" for a new direction we wanted to build was taken out from us, and our cooperation was stopped for a long decade.

Our second period of research cooperation, from the mid 1980s, was joint with Jean Bourgain. It also, I think, was very successful. That time I turned to the direction of convexity, but "asymptotic" convexity, not the classical one, and "pushed" Joram to discuss this subject during our summer stays in IHES. I hope he liked the outcome, as I liked it.

Our joint activities and cooperation were not reduced to joint research. From the start of the 1980s we organized a seminar (mostly in Tel Aviv) on Geometric Aspects of Functional Analysis, which soon became very famous and world known under the nickname GAFA Seminar. For many years it met regularly, generally twice monthly on Fridays, and attracted a lot of people from all of Israel (and many foreign guests). Six books of proceedings of this GAFA seminar were published during that time, mostly by Springer, jointly edited by the two of us. Later, the health of Joram did not allow him to come regularly, and the seminar changed its appearance.

In his work and his activity, Joram always emphasized non-triviality and difficulties, but also quickly caught new ideas and recognized tasty results. He did not allow "easy" works to come through his hands. This harsh approach of his kept the high level of research in Geometric Functional Analysis and, also, had a great influence on the Israel Journal of Mathematics during the period he was a leading editor.

The loss of Joram is a great loss to all of us, his colleagues, friends, and mathematics in whole.

(Vitali Milman, August, 2012)

PS (added in 2019) I should add that our long time cooperation was not without some controversial episodes, but I firmly decided to leave them out of discussion now and forever.

Alexander (Olek) Pelczynski (July 2, 1932 – 20 December 2012)

Three memorable moments with Olek.

It is a very sad and difficult time now. Friends, with whom I used to discuss Math, and asked for advices during my starting years in Mathematics leave us one after another.

During my years in Russia till 1973, I met Olek, I think, 5 or 6 times, when he visited Soviet Union. I would like to describe very briefly three of such meetings.

The first time it was around 1963 in Kharkov. Olek came to Kharkov to see Michael Kadetz. I had just finished University, and knew very little of infinite dimensional Banach Space Theory (my master thesis and PhD were on different subjects), but I already liked it a lot and my mind had been working only on its problems. Olek was the highest authority in the field, and knew the latest results from all over the World. He was our only contact with the World on this matter. I remember one of these problems, because I made a mistake related to it. For some reason (which reason I don't remember) I said that c_o space is not isomorphic to a dual space (because it does not have extremal points). Olek stopped me: "it means only that it is not isometric to a dual space and we don't know more". I started discussing, whether it could be corrected, but without any success. (Very soon later a very general statement had been proved by J.Lindenstrauss and Olek and became a very well-known fact).

The next episode I recall and wish to discuss happened around 1968 (and, of course, we, also, met during the Moscow Congress of 1966). I already lived near Moscow and I came to the Moscow University Housing to meet Olek. Among other things, Olek said to me that I had been very intensively using Dvoretzky's theorem, but it was not clear that it had been proved. "They", meaning him and Joram Lindenstrauss, see a gap in the published proof; it was written at one point "clearly", but not him and not Joram saw how really to prove it. So, he said, all your results in this direction are conditional. My reaction was to say that I did not know the fact, but I thought I, anyway, knew how to prove it using a different approach. "Then do it" - said Olek. This was the first "push" for me to prove it and to put it down in writing. Young person, as I was, would had never published already known result without such push (at least, such was my ideology at that time).

The third episode I want to disclose is personal, no mathematics involved. Once, around 70-th, Olek decided to visit me at my place, a scientific center near Moscow, Chernogolovka. This small town was absolutely closed for foreign visits, but Olek either did not know this or ignored it. And one day someone knocked to my door, I opened and saw Olek. I was very happy to see him (and we had two days of Math. discussions, as a result). My wife, Luda, ran to the only supermarket we had to find some food to prepare a dinner (Russian families did not have storages of food at their homes). Per usual, the supermarket had almost nothing to sell. She begged its director to find something, saying that she needed it for a very dear friend, who had just unexpectedly arrived. The director took pity on her, and took her to the main refrigerator that contained ONLY three mini-chicken that all looked blue. Of course, she took all three. After the dinner Olek said: "Your fish-soup was very good! From what fish did you cook it?" Luda was in tears: "Olek, this was a chicken soup" she said. But it, indeed, smelled fish, because these mini-chicken were fed by fish! Later, as we already lived in the West, we laughed over this story.

(Vitali Milman)

JEAN baron BOURGAIN (28 February, 1954 - 22 December, 2018)

Jean Bourgain, Summer 1999, Vancouver, my 60th   *Bourgain's Coat of arms*

On 12/26/18 I received the following message from Jean's sister, Claire Bourgain:

"It is with sadness that I have to announce to you that Jean peacefully passed away Saturday night. As you are one of his close friends, I did not want you to hear the news in an unpersonal way. We will terribly miss him. During the last years when he lived at our home in Belgium, he was always inspiring and kind, although being very ill

....

As we intend to have a strictly private and intimate funeral ceremony, may I kindly ask you not to diffuse the news until next Saturday. The official announcement will be next Monday.

....

Claire "

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I answered

"Dear Claire, I am in shock and could not move for some time.

....

He sent me his last paper just a couple of weeks ago. I feel so bad that I cannot write and express myself. Yes, we were very very close. I knew how he felt about things and events, and he knew how I felt. "

The last working table of Jean Bourgain

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When on Sunday, December 30, the information became public, I started to receive messages, phone calls and just conversations about Jean. And almost everyone started by saying "what a terrible loss for Mathematics!". Yes, of course, no doubt, but I felt terrible to hear it. I lost a very close friend, who was so bright and alive in my memories, in hundreds of episodes which are part of myself, my own identity, and, yet, people talk about the "loss for mathematics". I fully understood the decision to wait a week with the announcement, and was very thankful to Claire to have these few days only to myself, to overcome this shock, this loss for myself before I hear about the loss for Mathematics!

Thanks, Claire!

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The first time Jean wrote me about his health problems was on Nov 19, 2014:

"Right now I am in Belgium and will remain there for quite some time. In fact, confidentially speaking, I should update you about something not very pleasant. Basically, I am in quite bad shape as a way of putting it. A month ago I got diagnosed with pancreas cancer and presently undergoing chemo in my sister's hospital. This will be going on for several more months so surely in January I will still be here. Otherwise I'm in good mood because math is going well...... But, please, keep the above strictly for yourself.

Yours, Jean "

And later, after we discussed my visit to see him, on December 6, 2014 :

"Dear Vitali,

I just got the third chemo session. Usually they keep me 3 days in the hospital and then send me back home for ten days until the next one. But there were more side effects with the previous one, so things got delayed for a few more weeks then. So as you see, things are not too predictable. I hope that I won't be completely in the down period when you visit, we will see.

....

Jean   "

I first met Jean sometime in 1982/1983 at a conference. That time he was involved in all aspects of the classical (infinite dimensional) Banach Space theory. I noted his interest in finite dimensional talks and suggested to "introduce" him to this theory. A quick ironical smile passed through his face, but he came to Israel in the 83/84 academic year, first to Jerusalem and then to Tel Aviv to see me. We started our first discussion, and later published a paper (on "distances between normed spaces"). The next Academic Year 84/85 we spent together at IHES, Bures-sur-Yvette (near Paris). It was an amazingly productive time, many open problems were solved and we became very close friends.

 Later, during our scientific cooperation through our life, I was responsible, as an editor, for publishing 85 of his papers in two forms: 35 papers in the GAFA journal, and 50 more in the serious GAFA Seminar Notes (Israel Seminar on Geometric Aspects of Functional Analysis).

In my talk at the conference "Honoring the Life and Work of Jean Bourgain" on May 31, 2019, I shared some memories and reflections on Jean's style of work. I will not repeat them here - one can find the video of that talk at the IAS webpage. Many of Jean's expressions mentioned there became the standard "dictionary" for my students, who remember them and spread them to the next generation of mathematicians. (I had placed these Jean's expressions in the Chapter "Advices for students".)

Jean was a very nice and open person with the people he liked, and he had an extremely responsible and strong personality. When we (me and my wife) visited him in his sister's house in Belgium a few months after he was diagnosed with cancer, almost the first thing he said was: "Why should we be sad? Is being sad helpful? It is not, so we are not going to be sad!"   Saying this he brought a bottle of a very good wine, which he could not drink himself, as we learned after it was opened, and served it to us. He knew at that time, as did we, that the expected survival rate with his diagnosis was between half a year and one year! It was a miracle that he survived and worked(!) four and a half years after he was diagnosed. The creator of this miracle was, of course, his sister, Professor of Medicine, Claire Bourgain. We are infinitely grateful to her.

The very limited space I was allowed to use in the article I copy here allowed me only to recall a couple of stories that may demonstrate Jean's character, his human side, so, I will add to them a few more.

Jean mostly worked during the first part of nights. This was his way to be isolated, undisturbed, having absolute quietness. However, he also needed, from time to time, an atmosphere of "white noise". For that he would often take a metro from Bures-sur-Yvette to Paris (about 40 minutes) and would immediately return. Often, for the same goal, he walked along the Champs-Elysees.

Jean, actually, liked to feel being in extreme situations. Once we flew together from the US to Germany to an Oberwolfach meeting. He did not, yet, have a driver license (which he received quite late, around the age of 33-34). Jean suggested to rent at the airport a very good car, like a Mercedes, and wanted to be driven to Oberwolfach at the speed of 200 km per hour (which is allowed on the highways in Germany). "Can you do it?" - he asked, and I reacted "of course" (I am not proud with myself about this). So, we drove. Jean looked hypnotized at the speedometer all the way, without moving.

Jean hated to teach classes, but liked to give scientific talks about his very latest results. Once he told me "two months remain to the conference (to which we were going together), but I did not, yet, prove a theorem I plan to present. Actually, I have not, yet, decided, what theorem I want to prove". (Of course, he, eventually, gave a talk about the results he obtained in the last month before the meeting.) However, about regular classes, even for faculty, the situation was different. Perhaps, he just could not understand, what he should explain in more detail, and what is obvious or known. One funny story gives the picture. It was the Fall of 1985, Jean just started his job in IHES and also received the Doob Chair in Urbana-Champaign. By the regulations of the Chair, he had to start with some classes, which were a series of lectures (for faculty). He asked me to come, to serve, I believe, as a moral support. His lectures were twice per week and I arrived for the first week. We had coffee before the first lecture (lectures started, as far as I recall, at 12) and Jean intensely computed something on a piece of paper. I asked: "are you computing something for your talk?". "No" - he answered- "I am computing, how much I will be paid for the lecture, and, when I see the number, I

feel easier about delivering it". I understood, and, in fact, found it reasonable. In two days' time, Thursday, we again had coffee in the same place before his second lecture. Again, Jean was computing something. I asked him "what are you computing now, preparing your lecture?" "No" -Jean answered- "I am computing, how much I will be paid for the lecture. And when I see the number I feel more comfortable about going to deliver it". "But you did it already on Tuesday" - I reacted with some surprise. "Yes" - Jean said- "but I need to SEE it to be able to give a talk". I think this explains well, how he felt about giving classes.

Generally, in his young years Jean's talks have been considered to be non-understandable. I think that he just did not put an effort to realize, what the audience may not know. This worried me very much during the 84/85 years at IHES. Our joint dream was that he will impress the permanent members there to the level that he will be invited to stay at IHES at least for the next two years. In the evening before each of his talks there (and Jean gave a lot of those) we discussed it. I, usually, would tell him: "When people at the conferences don't understand, how you solved problems they worked on for years, they respect you even more. However, if the people at IHES, like Gromov, Sullivan, Connes and others, will not understand you, they will consider you to be an idiot, not themselves! So, if you want to impress them, they should understand!" And Jean tried, and succeeded! In June 1985, he called me to Kiel (I was already in Germany) and said: "Kuiper (the director of IHES) offered me to stay at IHES". I asked with excitement: "For two years?". "NO"- Jean answered- "permanently". "And you agreed" - I noted. "Not so quickly!" - suddenly reacted Jean. "Why? what, if they will change their mind?". And then Jean explained to me that from the moment IHES issued the invitation to him, they became more interested that he accepts it, than him to accept. So, he added, I may discuss the conditions! And he, for example, among other things, gained the increase of his salary; these were very low in France, including IHES. And this increase was, of course, applied to all permanent members, including Gromov (who never discussed his salary). Unfortunately, Jean stopped his effort to be understandable, and his talks later became again difficult to follow.

However, in 2016, his video talk at the conference "Analysis and Beyond: Celebrating Jean Bourgain's Work and Impact", his last, I think, public talk, was truly excellent, extremely well prepared and delivered. It was such joy for me to hear it, and then his clear, short and up to the point answers to some questions from the audience. Only a few people in the packed auditorium knew that just a day earlier he had a very heavy (scheduled) chemo session. And under these tough conditions he delivered what I believe was his best lecture ever!

(Vitali Milman)

Misha Gromov and Vitali Milman, around 1976   Misha Gromov and Emanuel Milman, 2019

Many years later we discussed with Misha Gromov that year in IHES, and he told me: "I saw, Vitali, how you acted during that year to push Jean into IHES; I did not want to tell you this back then, as decided to be neutral". Actually, he did not see the whole picture, because at that time I acted for both Jean Bourgain and Gilles Pisier. During the first semester I did not, yet, have made my decision on who I preferred. But Gilles, as I had learnt before the summer, had huge family problems during that period. So, his activity and concentration were low and it was easier for Jean to go ahead.

I will talk a little bit about Jean's Fields Medal. He received it in 1994, and getting it wasn't going very smoothly. I did not hear from anybody that anyone at the Committee objected him getting it based on his scientific research. Under discussion was his age and what should be the rule. I think the reasons behind were different, but formally, the objections were expressed through discussion of age. The formal written decision was that the candidate should turn 40 after or January 1 of the year Fields is awarded. Jean's birthday was February 28 and he could receive it. In order to show how difficult it was for the committee to come to this

decision I should draw your attention to the timing of making it. Usually, Committee makes decisions on Fields medals during the month of January (of the corresponding year). The decision very seldom goes beyond this month. But in 1994, it was March, when Gromov, who at that time attended the conference in Columbus, Ohio, which I too attended (it was the conference due to the opening of the new Math Faculty building), when he received a FAX from one of the Committee members with a request to urgently write another supporting letter on Jean. "There are again problems" - was an approximate text of the FAX. Misha sent one page. The idea of his text was: Jean worth three Fields Medals, but you are to give only one (of course, this is my approximation of the text). It seems to me that on that last meeting the decision was, finally, made. At least, leaving in April to China, I firmly knew that Jean will have the Fields Medal. Yet, when I was back in the last days of April or in the beginning of May, and Jean for some reason called me home, I asked him, whether the information on medal has arrived to him, and he answered "No", but later through that conversation he pronounced: "Stop, Vitali, some letter has arrived, let me open it", and then I heard: "Yes! this is about the Fields". That much it was delayed!

During Jean's Princeton period (it started in 1994) we, still, met, but, naturally, less often. He always came to the conferences I invited him to; he, also, came to Vancouver for my 60-th, and to Tel Aviv in 2009 to my 70-th anniversaries. And during every my visit to US, on my way out I would make my way through Princeton to see him and discuss life and Mathematics.

It happened once that I arrived exactly, when his mother, Margot, whom my family knew very well, passed away, and he left, of course, to Ostend for a few days. It was a very difficult and sad period for him. His wife Mei-Chu Chang, wrote to me later: "hi Vitali, I didn't have a chance to tell you that after you talked to Jean (after his return from this sad event) his tone of voice changed, and he started to lighten up and smile etc. what a charm!"

At the memorial meeting in Princeton on May 30-31, 2018, a few months after Jean passed away, his secretary from 1994, Ely, approached me to say that she observed Jean all these years, and he was changing, when I was passing through IAS. "You were like a father to him" - she said.

I still don't feel he left forever. I still feel his presence, and I see him quite alive. There is his portrait on the wall of the living room in my apartment in Herzeliya, so, I can see him every day.

I will end this my very short piece of memories of Jean by explaining how we worked together. With Jean enormous technical abilities and very quick understanding of Mathematics it is a question to be asked what was my role in our joint papers. Actually, he formulated it once himself. This was in the same year 84-85, which we spent in IHES (in Bures-sur-Yvette), the most productive year in my life, as I described in "To-day I am 70" and have already mentioned here. Here is the story.

Once we discussed some problems and the next day I brought to Jean a few pages of computations answering whatever was discussed (I completely forgot now what it could be). Jean took these pages and put them accurately to trash. He said: "Vitali, I know to do computations better than you. I expect from you the ideas and a general picture of what is going on, some geometric insight into the problems [he meant in our analytic problems]. Then" - he continued - " I will compute things myself. But your computation I will not even open" - concluded Jean. Then he added: "I suggest a bottle of Champagne every time you will improve my computation" (this was young, very competitive Jean). Because of what happened next, I like to joke here that Lord had heard this remark. During that week (the story started on Monday) I improved Jean 3 times.

The first was a misprint, which changed the outcome, but Jean said he counts it.

The second was a peculiar case. We walked through the park of IHES, and Jean asked me some question. He could not see a geometric picture behind the matter, and dwelled very intensively on it. I did not understand his question quickly, started to ask my counter-questions and requested the matter, and dwelled he explanations. I must had done it quite properly, as in the end Jean solved the question exactly, when I understood, what he was asking. He considered this to be a second bottle (he was happy to solve the problem).

But then the third case, the serious one – I have a published paper on this subject. This is about a Low M* estimate with a linear estimate on a proportion of co-dimension in the formula. Just a couple of days earlier Jean told me that he knows to prove a sub-exponential estimate. My original result (a year earlier) was exponential estimate. I did not care at that time to have a better estimate. It was not clear that a better estimate was needed for anything. But our joint with Bourgain work on "reverse Blaschke-Santalo inequality" showed that it was worth having at least polynomial estimate. And I proved linear a day or two after Jean had sub-exponential (I never saw Jean's proof; I think, he did not write it down). Jean could not believe it. He, as I realized, thought about it last days. He wanted to read the proof, which I had already written down. It took him a couple of hours, then he came to my office and said "Yes". Actually, at some point he even produced a

better argument. And then he added: "I terminate my suggestion to bring a bottle of Champagne after every your improvement of my computation".

The fun is that I don't remember a single more case of me improving Jean's computations later. I joke that Lord taught him a lesson, and withdrawn his attention from me.

....

Next, I would like to bring in some of my memories about the great mathematician Fritz Hirzebruch. This was written on a different kind of occasion, then my memories above. Professor Winfried Scharlau (who received PhD under Hirzebruch's supervision in 1967) wanted to write a book about him, and asked his friends to send him their memories. Hirzebruch's wife, Inge, provided a list of whom to ask, and I was on that list. Indeed, for some while we were quite close, and, so, I was/am happy to write about this remarkable person.

Hirzebruch, Friedrich (17 October 1927 – 27 May 2012)

Professor Friedrich Hirzebruch was one of the nicest person I ever met. Straight, clever in everything he did or said, obviously very honest. To talk with him was a pleasure, and he expressed his thought in a very clear way. This was the case with his mathematical lectures, but also with any other subject we discussed. I met him and his wife, Inge, first time in Israel in the Summer 1981. They arrived by the invitation of Ilya Piatetski-Shapiro. At that time, I had been helping Ilya in many things he was involved in, helping him to host Hirzebruch family was one of them. Of this hosting the most memorable part was our visit to Eilat. It seems to me, the Mostow family was there with us, as well; I am not sure, though. Sinai Peninsula was still under Israeli control, and we drove about 70 km from Eilat deep inside Sinai to one very beautiful coral beach. Professor Hirzebruch and Inge were overwhelmingly happy! Indeed, being there was like spending time in Paradise - quiet, peaceful and indescribably beautiful. Seldom, I meet people in this happy state of mind, as both of them were there those few days: evenings in some city bars, and during the daytime driving around. I think, all his life he remembered that trip.

Through that visit Prof. Hirzebruch had definitely developed a very positive view of Israel, and he liked visiting us. I remember so many of his visits to Tel Aviv, Bar Ilan University, Jerusalem, that I cannot count them, so, I choose to briefly talk about two of them.

In 1988 Prof. Hirzebruch received the Wolf Prize awarded by an international Committee based in Israel, and he received it from the President of Israel at the ceremony held in Knesset. I was present there, as well as at the following reception and a few formal parties that accompanied this very important event (like the reception given by the Ambassador of Germany). At that time the Wolf Prize was the most important event in the World of Mathematics, which was equivalent to awarding Nobel Prizes in the Worlds of Physics, or, say, Chemistry (by the rules set by Nobel, the strictly mathematical research is not eligible for being awarded the Nobel Prize). So, during the ceremony in the presence of major Israeli and foreign dignitaries Hirzebruch approached the podium and said the following (I am presenting his short speech by memory, in my own words):

"I am a professor of Bonn University. Issai Schur and Felix Hausdorff were professors of this university. Schur emigrated to Palestine in 1940 because of Nazis persecution, while Hausdorff committed suicide with his wife in 1942 not to be sent to a concentration camp. The memories of them are with me, when I walk in Israel". This was said with such emotional strength that silence and tears accompanied it in the audience.

Around a year after the Berlin Wall fell and Germany was united, Hirzebruch visited us and came to me with a request. (Something needs to be explained here. There are very few purely research positions without teaching in Mathematics in the Western World. Say, till very recently, IAS, Princeton, was the only such Institution in US with around 7 permanent positions. In a couple of more such Institutions in US the Director is the only such position. There was only one such Institute in the West Germany, the Max Planck Institute for Mathematics in Bonn, and at that time Hirzebruch was the only permanent researcher there (as its Director), yet, he also held position at Bonn University and taught there. Later, when he had to retire, he appointed a couple of people to continue taking care of the Institute. However, the situation in the East World, including East Germany, was different. Actually, hundreds(!) of mathematicians in East Germany worked in different (so called) research institutions without any teaching responsibilities.)

So, Hirzebruch came to me with a long list of names of the East Germany mathematicians (or, better to say, people occupying positions called "mathematicians"). I don't remember the exact number of them, but it could have been even thousand (perhaps, it was my feeling and it was only a couple of hundreds). He asked me to mark any name I know on this list, or even not really know but only heard about. And he added that he would not ask for any recommendation, it would be enough that I had heard about them.

"We cannot support for a long time so many people", he said," but I may try to support around, say 40 - 50 of them, and I am searching for the names of mathematicians, of whom the mathematicians I trust even just heard".

He added that he himself cannot collect even so small number of names from this huge list. He, Hirzebruch, was made responsible for all Mathematics of Germany (both, in some official form, but also, and most importantly, he felt this was his moral obligation). I had chosen for him a few candidates from that list, but the number was not enough to come even close to the numbers he was ready to help.

In my last brief set of memorial stories I want to write not about Hirzebruch visiting us in Israel, but rather about the visit of my family (my wife, my two small children and me) to Bonn. It was during the summer of 1985, and he hosted me in Max Planck Institute for Mathematics. There were many bright worth telling stories during that month, and in there I had finished and put down in writing one of the most significant results of the Asymptotic Convex Geometry, the reverse Brunn Minkowski inequality, called to-day "the result about the existence of M-position", which I mention, because Hirzebruch was very much keen on everyone he invited doing Math during their stay there, which, actually, was what pushed me to prepare this result as a preprint of the Institute - Hirzebruch was happy to hear that my visit was a scientific success.

There was, also, something funny that happened during that visit. It was July, and IHES had just made a decision to invite Jean Bourgain as its permanent member. This wasn't, yet, formally announced, but Jean let me know over the phone. So, once during the lunch in the Institute, Hirzebruch asked, whether anyone heard, who's taken to the IHES. I said: Bourgain. But, since Jean was young and in different, then Hirzebruch, field, Hirzebruch was not, yet, familiar with this name. So, he asked me who Bourgain was. Being in the mood for joke, I answered: "Belgium; Belgian Pierre Deligne left (he left for Princeton), and Belgian Bourgain is taken". In response there was silence.

My joke was not understood. Yet, in two days I, again, had lunch at the Institute, and someone, again, asked whether it is known who is being taken to IHES (nobody said it, but, somehow, it was in the air that this was about replacing Deligne). Then Hirzebruch answered: "Bourgain, Belgian left, Belgian is taken", and he laughed.

By the way, by the request of Hirzebruch I invited Jean to visit Bonn and he came for a few days and delivered Colloquium in the Institute.

We once had our joint visit to some famous restaurant on a hill not far from Bonn. Both our families were there. Fritz told us that many years back, Helmut Schmidt, the Chancellor of West Germany hosted Brezhnev, the ruler of the Soviet Union, in this restaurant. Brezhnev liked collecting cars, and the Chancellor gave him a present, a Mercedes, which Brezhnev wanted to try immediately. So, he drove down the hill. Perhaps, he was already drunk, or just a bad driver, but he had an accident and hit the car. This significantly spoiled Brezhnev's mood, but Helmut immediately presented him with another Mercedes. This made Brezhnev very happy, and Helmut could achieve through this meeting whatever he wanted.

Our two children were with us, and Hirzebruch invited our youngest daughter, Anat, who was around 7 back then, to dance. It was such a nice and cute gesture, and the visitors of the restaurant, and, of course, us, enjoyed it very much.

Professor Hirzebruch, obviously, loved Israel, liked visiting us, and helped us at every forum, where he was present (he always knew very well how to do this). Here's one example: he was absolutely instrumental in the decision of the European Mathematical Union to allow Israel to joint this Union, and, as one may guess it, this could not be a trivial exercise, since many influential people, for example, Atiyah, objected. Being accepted into this Union was extremely important for Israeli mathematicians, especially the young ones - this allowed many of them, for example, to receive the European Prizes for young mathematicians. Hirzebruch was, by the way, the President of the European Mathematical Union during that period.

Another example of such enormous help was the establishing of a few Minerva Institutes of Mathematics at Israeli Universities. These are institutions that are based on support of Germany, and which promote joint scientific cooperation with Israel. During decades these Institutes supported our joint research with Germany's mathematicians, supported the Conferences and our students.

Thank you, Fritz!

(Vitali Milman)

4. TIME OFF

Until a few years ago any my vacation was, actually, a pause for me, a short period between conferences or change of places of scientific visits. And it was not a real time off for the brain.

But in recent years things have changed. Less and less amounts of obligations, my last PhD student is for a few years, already, independent, the direction of Math we were developing with Hermann Koenig ended in writing a book (which is already out there). I, still, have one direction of my high interest, which started from a joint paper with my son Emanuel and my last PhD student Liran Rotem, but neither one of them is interested in continuing this topic, although I think it may be of a huge interest for Mathematics.

Still, GAFA occupies my time, since Leonid Polterovich, who is supposed to pick it up from me, still, wants me to be involved. But this may too soon end for me.

Yes, the next year I will have two postdocs and we, jointly with Apostolos Giannopoulos and Shiri Artstein-Avidan, should work on the second part of our book with a hope we will finish it through that year. We had such hope for the last 4 years, though. And the postdocs are of joint responsibility with Shiri, rather, than my personal. So, suddenly, there's lots of time on my hands for relaxing, for doing nothing. I will remark that nothing to do is, perhaps, not so difficult. But to do nothing is, very much so. And I think I am mostly involved exactly in this: I do nothing.

The last 10 years we visited Hermann Koenig in Kiel very often, at least once per year and even often twice. It started with the Humbold prize I received (for the life-time achievements), but then it continued because of scientific intensity of our joint work. We wrote in this period 15 papers and collected this knowledge in a book "Operator relations characterizing derivatives".

The University of Kiel (very old University) has a guest house just in the Port. We always stayed in this House. Between it and the sea is at most 20 meters and only walking area. All kind of Ferries and Cruise ships park meters from our house. And if a few cruise ships are coming the same days, then just in front of our windows there are settled windows of the cabins of a ship. This atmosphere of cruises is very attractive, especially for me, who was born on the seashore (that was the city of Odessa on the Black Sea). So, gradually, we got used to it, to the cruise atmosphere! Now, we take cruises at least once per year, but often, actually, twice. And this became our real time off! To the first cruises I, still, took with me some research papers to read or articles to write, but not then. Now this is solely the time for relaxation, for "nothing to do". Although, writing this essay contradicts my statement - I am on a cruise right now, in my cabin, lying and writing it. Oh, well, writing down your memories is not difficult, it provides a true relaxation. I find pleasure in seeing that I did something with my life!

Luda, my wife, has a few more ideas on how to relax, how to take time off. We will adopt these ideas soon. But I feel I, still, don't know how to relax, to take a vacation, so, I have nothing to write in this chapter!

PS. One interesting story should accompany it, though. During all my life from childhood till 60 + I could not be on boats and ships. Even in a car I often felt the so called "sea sickness". It was absolutely terrible, I felt badly almost immediately on a boat or ship.

But then I had a very bad car accident, I was almost killed. After a week in hospital and a few months of recovering we went on the Sabbatical to New Zealand (and to Europe and US after that). I could hardly walk and used taxi even for 200-300 meters. However, many things in the activities of my body changed to the better. In particular, my sea-sickness passed completely, absolutely. I now just don't feel rough water, may use boats, ships, never feel bad, when someone drives me. There were other positive changes. Perhaps, the most interesting one (and, also, the important one) was the unbelievable fact: my brain started working better. This is the proven fact. I easily gave proofs to some facts that I attempted to work on earlier, but failed. This feeling of much better and clean brain work I had for many years after the accident. Actually, Misha Gromov told me that it is the well-known fact, and explained the reasons behind, which I remember, but I am too lazy to write them down.

5.   My involvement in Medical & Biological Research

The Medicine and Biology as a science attracted me already in the beginning of 60-th. I think, the fashion among mathematicians for medicine was triggered and pushed forward by I.M.Gelfand. His youngest son was diagnosed with leukemia before he turned 5, and then, I think, Gelfand organized a chunk of medical World in Russia to work on leukemia. He started running a regularly working joint between Mathematics and Medicine seminar. Unfortunately, his efforts did not help his son, he passed away - time for successfully treating leukemia wasn't, yet, there. But the fashion for medicine spread among mathematicians. Joint between mathematicians and medical science researchers seminars started running in many places. One of them proceeded in Kharkov.   So, in the beginning of 60-th, there in Kharkov, I had my first scientific contacts and even cooperation with medical researchers. We studied Thyroid and related to it sicknesses. And I, actually, made some discoveries (not published; "too simple mathematics" - I concluded). I was told later that

they were used with references made to me, to these unpublished results, even so in a Doctor of Medical Science Thesis of some well-known Professor of Medicine.

Some number of years after our moving to Scientific Center in Chernogolovka (near Moscow), my contacts with researchers in Medicine were reinstated. They were diverse, numerous and led to some funny stories (I will omit them here). Here I will concentrate on one of those contacts, the one that had lucky continuation some 30-40 years later in Israel. I mean my contacts with Elena (Borisovna) Vladimirskaya, Lena, as I called and call her now. Everything was remarkable around this young woman; in particular, her devotion to both her family and science (such devotion is, probably, less common within Medicine, than within Mathematics).

Elena Vladimirskaya, 2019

Her father was director of "Melodia" firm, the institution, under which umbrella all (without exception!) Russian musical recordings were produced. Of course, to Western people the understanding of the level of powers such a person possessed is near to impossible! The intellectual production was very limited in quantity, while it was in a huge demand. I liked him very very much, and I think, he also liked me. It means I could have any industrially produced in the Soviet Union music, but, also, new books, if he already had these from the previous production, and, if we had money to pay the nominal price. (To emphasize the problem, such very much in demand book, as "Joseph and His Brothers" by Thomas Mann was published in just 5.000 copies, hardly enough for all the libraries in the Soviet Union.)

My wife, Ludmila, and I could come some time before Lena was back from her work (for us coming to them meant taking bus - some 45 km from Chernogolovka to Moscow, and then within Moscow switching to metro (subway)); her father would sit with us and entertain us with stories of his rich life experience. I had learnt later that even Lena did not know some of them, while they are worth to be well known. Such a pity I remember only very few of them. Here's one of them that, actually, had significant resonance in the Soviet Union, but nobody I ever met knows the truth behind it.

The preliminary information is this. In 1953 died Stalin, but until February 1956 he continued to be treated by the Soviet propaganda, as a great state hero. His status of absolute dictator, naturally, implied that his name was mentioned in the Soviet national Anthem. Every midnight the state radio played it, both music and lyrics. One day around 1955 the text was dropped, had been played only music. Immediately, whispers spread - Stalin's name wasn't mentioned! Something must be going on! However, the reason was quite "prosaic". And here comes the story.

One early morning a few KGB agents knocked at the door of Boris Davidovich Vladimirsky's (Lena's father) apartment. That wasn't the first such event. For anything that had been "industrially" happening that involved music, he was considered personally responsible. These time agents informed Vladimirsky that previous night the lyrics of the Anthem had been somehow changed. But this was impossible, because the recording was performed once forever - no single word could had been ever changed. At this point I have to switch to Russian, and then I will explain the meaning.

Here's the quote in Russian: "Souz neryshumui respublic svobodnyh splotila naveki velikaya Rus'..." The meaning is that the great Russia solidly consolidated together the free Soviet republics. But the agents claimed they received a letter from somewhere that one word had been changed, that "splotila" (consolidated) was changed to "shvatila" (forcefully grabbed, like by their necks). Carrying so drastically different meaning, these words phonetically sounded very similar, actually undistinguishable. Clearly, this was the top scandal!. Yet, of course, nothing was changed in the text, instead, simply, as sung and transmitted by radio these two words were easily confused. Vladimirsky immediately understood this, but decided that the agents should come to this conclusion themselves. So, next midnight the powerful chorus of the Soviet Army sang it and was newly recorded, high level Committee was present, and authorities' representatives were sent to different places of Soviet Union to listen how the Anthem will sound on radio. The collected data gave 50/50 result. One half heard one of the two words, another half - the second word. Officials didn't know what to do, so, by the end of the second day, Vladimirsky suggested to temporary, "ONLY temporary", for the coming midnight, play the music of the Anthem without lyrics. This was approved, only from that night on any different decision wasn't made, lyrics was never again played with the Anthem!

But let's return to medicine.

I remember some of our discussions with Lena, and how I understood something about anatomy of the blood vessels by just analysing the distribution of erythrocytes in blood. Of course, all this was well known, but experimentally, through numerous surgeries. When we met already in Israel in around 2009, interestingly, we remembered different things from that Moscow period. Lena told me that some of my advices made before I left Russia (in 1973) were used in research of her group; some formulas I wrote for them were useful and were used. She told me that I didn't want to publish them or rather to join signing the publications, reasoning, again, those were the results with the "too easy mathematics". I had no recollection of that. But she did not remember anything about me learning from her all that stuff that she taught me.

We were happy to meet in Israel. Lena emigrated a few years before we met, although her son lived already for decades in Israel - Lena's husband was too an important figure to be able to leave Soviet Union (he was a military captain of the first rank, one rank before admiral), and, so, she emigrated after he passed away. Her own status back in Russia was also very high. She, a professor of Medicine, and a personal doctor of Raisa Gorbachev (Soviet Union's "first lady"), held position of the vice-director of the most important clinic in Moscow for children with leukemia, aside from a number of other important positions. Yet, all of that was less important for her, than being with her son, and, consequently, she emigrated. In Israel she attempted to find me, and someone told her that I was in US. Perhaps, that person meant that I was in US at that particular moment, for some short period, but Lena understood it, as I lived there. Somehow, by incident she came across my Web home page and email address, thus, realising that I lived in Israel.

So, at her first dinner with us in our Ramat-Hasharon apartment, I inquired and Lena updated me on her medical research; my interest had been highly elevated by what I heard. And we again started to discuss medicine. I had accumulated enough of resource, which, luckily, after my retirement I could spend on any science activities I wanted to (not just Mathematics), and I could pay for some experiments Lena suggested we may perform to check and confirm some conjectures we had about effects Lena observed in her work. It is, by the way, amazingly interesting to see the results of experiments. Regardless of whether they confirm or not your expectations. They always contain miracles!

I slowly developed the following, very rough, analogy. Assume some forces outside us perform some experiments on us, people. If they will see stable repetition of the same reaction, it would mean that they came across some "reflex" (I would even call it an "absolute reflex"); say, when a medical hammering instrument hits our relaxed knee at certain specific point of it located slightly below the actual knee bone, our so far freely hanging leg jumps. This is the experiment that may be repeated. But this is a less interesting thing! If the experiment touched our consciously made reaction, then different people would react differently! Even the same person, then, may react differently from one run to another run of the experiment. Thus, then one cannot repeat identically this experiment, it is not repeatable! But at the same time, one may, consequently, observe highly interesting reactions. In order to expect receiving even some roughly integrally similar reactions we should desirably know much more about the species (e.g., about the cells that we may involve in our experiments).

Let's consider another example, not really good and not very intelligent. Again, imagine someone experiments on us, people. They through a truck of oranges onto a street of Tel Aviv, and then they observe, how quickly the oranges will be picked up by the passersby. Well, they will have to wait for long, very long, if, at all, anybody will take any. But now the same experiment performed on a street of Moscow (especially, in winter time) will show that all oranges disappear very quickly.

I seem to be entering the world of fantasy speculations, although, if someone wants to be nice to me, s/he may call it the World of Conjectures.

So, for a moment I will return to reality and say that Lena and I had published jointly 2 or 3 papers (depending on how to count, because one of them is the translation into English of a Russian one; so, in this sense, this makes 2+1). They are about communication among cells. Roughly speaking, before the cell-phones were invented the whole world of medical researchers and biologists was sure that all information/communication between cells happens through contacts between some molecules and receptors of cells. And, indeed, such contacts were observable. But we had put forward the possibility of the not direct contact communication/spread of information, similar to how human contact with the use of cell phones. We discuss some effects that may be observed and explained only through this way of communication.

And I will now more substantially enter the World of Suggestions/Conjectures.

6.   LIFE and CONSCIOUSNESS

Whatever I had chance to recently read on cells leads me to the following (conjectural) principle:

LIFE = CONSCIOUSNESS

(i.e., there is no living object that does not have its own understanding/consciousness).

Of course, consciousness of a specific form of life may be drastically different from what our consciousness may imagine. A lot of parameters influence it. Say, the period (timing) of the life (from possible minutes till hundreds of years, or even much longer); the level of independence (from being a cell of multicellular body, or a life in a large community, to relatively individual life); the level of dependence on other kind of living being; the form and way of multiplying themselves, i.e., reproduction; and many many other parameters.

But consciousness may also "get elevated" growing into a "consciousness of a city" or "consciousness of a country", also, a consciousness of some group of individuals. (During my time in Israel I observed that the consciousness of the Department of Mathematics of Hebrew University in Jerusalem is very different from the consciousness of its many individual members; when the same person represents the Department, this person is ready to do things, much less honest, than I could ever expect from the same person individually.)

Then among others, a very important kind of consciousness to consider is the "consciousness of the species", or sub-nations of the species (we can easily observe this kind of consciousness among humans). That is, consciousness may be changing its "societal" levels down to the level of cells, microbes, viruses and, perhaps, even lower.

I plan to offer some examples, but I will first demonstrate one case of miraculous cleverness of a cell within multicellular living being.

One of the most fantastic discoveries made some 25/30 years ago was the discovery of APOPTOSIS. Again, all of my knowledge about it comes from our discussions with Lena. So, apoptosis is the program inside a cell which is activated to kill it. That is, a cell may receive an order to die! This, of course, is a necessity based on survival "instincts" of a multicellular body, e.g., such is a protection from a quick cancer related death, which will most likely occur, if cells are allowed to perform uncontrollable divisions. In fact, the regulation of how cell dies should be very strict. Fantastic thing is that such program is, actually, permanently activated in cell, it is always ready to act. The only way for a cell to continue living is in performing certain job, which it is supposed to keep performing. Then the action of apoptosis is being delayed up until the cell is performing something else.

I will now illustrate how clever a cell is by demonstrating some of its actions. Sure thing, I will simplify the reality. My apologies for this. So, on its surface a cell has numerous receptors, they may account, probably, for hundreds of thousand, but also may count below hundred. To help us see the picture, let's imagine a cell rescaled to the size of a town like Ramat-Hasharon is (I lived in it recently). Then the receptors would be some structures of the size around 3 to 5 floors on its side surface. (Let's remember that cells are 3-dimensional, not 2-dimensional, like we may percept a city on the surface of Earth.) Then let's imagine a molecule approaching the cell. It contains some information and, maybe, in it is an order for the cell to carry out some action. It may enter the cell ONLY through these constructions-receptors (whether this is a physical intrusion, or some message is being carried in). Relative size of the molecule in this chosen scaling would be as a person.

Not every receptor is ready to accept every molecule-messenger - receptors are molecule-specific. And there may be no receptors at all on a given cell for some molecules. So, these ones are not allowed in and, so, they do not carry any deliverable to this cell information. But let's assume, that there is a receptor, and a molecule arrived to the receptor that is suitable for it. Now the receptor should carry a decision to let it in (molecule or information).

Stop! not so quickly. One single receptor will NEVER carry a decision. It will either call a similar receptor located not far, so, that this receptor may move to it, to the original receptor, or it will create near by the same kind of a receptor (double itself). Now, these two receptors will jointly carry the decision.

Again, Stop! Not so quickly. Such a schema is for some "simple" decisions. If the molecule carries such a crucial order as to start the apoptosis, then two receptors will not be considered good enough. Depending on the form of apoptosis there will be needed at least three receptors, but may be even 6 of them. Speaking roughly, there are two types of apoptosis: a slow one, which will last many hours, and which may be stopped and reverse during its development, and another one, very quick, immediate apoptosis, which after its start cannot be stopped. The first, the slow one, is initiated inside the cell. It is exactly like this, the inside the cell order to die, to commit suicide, due to something wrong that happened in the cell (the risk of uncontrollable division is the highest danger for the body, in which this cell is an integral part). This is a very interesting process, which I will explain, again, in a very simplified way. There is certain gene in the cell that "observes" the scene. If it is noted that something is wrong it immediately stops all activity of the cell letting it to repair the system and return to normality. If this does not happen during some period, this gene (experts call it: the

"night guard") activates the family of genes (which I will call "jury", as it plays this role), all together 16 such genes, 10 of them are always pro-apoptosis (let's associate with them "+" sign), and 6 are against apoptosis (I will associate with them "-" sign). These genes produce some molecules that are involved in some activities that end in joining these molecules into pairs. There may be pairs of (+,+), or (+,-), or (-,-) types, although, some may remain single. Now, happens computing of "votes", through which the "lonely" (not paired) molecules are not being computed (they "did not come to vote"), just as well as the "indecisive" (+,-) pairs don't (it is abstained vote), while, of course, (+,+) is being "voted" for, and (-,-) against the apoptosis.   The majority decides the fate of the cell. (If I am asked regarding what happens, if there is an equality; well, I don't know! But I suspect it is the same as no apoptosis vote). There may be noticed more substantial problem: the a priory prevailing of the pro-apoptosis molecules, one may think, may mean the predetermination the pro-apoptosis decision. However, there is another parameter involved. These genes (16 genes of the "jury") produce molecules with different ability to join another molecule from this family. Every of them has regions responsible for the ability to join another molecule. Among 6 of anti-apoptosis ("-" sign), 4 have 4 such regions, and the remaining two - 3 regions. However, among 10 molecules of pro-apoptosis ("+" sign) only three have 3 regions (no molecule has 4 regions!), one more - 2 such regions, and remaining 6 have each only one such region!! So, they have very weak ability to co-join. And this creates the balance!

If the apoptosis is chosen, the other gene is activated, and a very interesting next step starts: the real gene-"killer" is activated, which does the job (also very interesting). However(!!), there are some proteins that may block the action of the killer. For the whole body of cells this is a very bad sign, since in this case the "unleashed" cell will start uncontrollable division (cancer). I will drop this part now. But what will happen, if the made choice was against the apoptosis? Then the cell continues performing its job, the one it should be doing and was doing, when it was stopped, and everything then looks normal, yet, our major "judge", the "night guard" may restart the process and call again for the above described family of 16 for making one more "vote", if something, still, worries it.

Quick apoptosis also may be stopped, but on a very preliminary stage of its development, the stage, when the order just came in. I am not sure whether this is also programmed in the incoming order that the cell receives from the outside. So, not having any scientific justification, I will call such quick apoptosis "(a)-apoptosis", while the unconditional order to die - "(b)-apoptosis". Maybe you have already guessed what I am going to say now. I don't know whether this is, indeed, so, and/or maybe it is already known to be true, but I think that for an (a)-apoptosis to start it is enough to have the decision made by the only 3 receptors. But for a quick (b)-apoptosis 6 receptors are called to jointly decide!

And in any case, wouldn't you agree that cells act in much more clever way, than we, the humans, do?

I think I should feel pity for you, if you, still, don't note "consciousness" in such behavior.

I will provide some additional information later. But now I would like to change "the scale" of our discussion and discuss "consciousness"/"thinking" of some huge living objects that even someone very much "pro"- my general philosophy may not expect. I will talk about trees. Of course, in general trees are to such an extent different species that we are unable to recognize their traces of "thinking" or "consciousness". But there are two very different kinds of trees that both make these signs readable for us. I personally had chance to observe both of these kinds, one in the jungle of Amazon, and another - in Cambodia. Perhaps, there are many more, but I had observed only these two.

One of these kinds is a "walking tree". These are the trees that move (walk) along Earth surface. Not quickly, around, say, 5 meters per year. Whoever did not know this and does not believe me, search Google for "walking tree" and see hundreds of pictures, including those that show the very process of this "walk". By the way, these are very big and high into the sky reaching trees with large trunks. Around 2 meters from the ground, such trunk is being divided into lots of "branches" going down. One may think of them as roots that hang a couple of meters above Earth level. So, to move in specific direction, a tree sends from its trunk above Earth new roots in the chosen direction, and, when these roots firmly settle inside the soil, some roots from the opposite side, the no more needed ones, die, and, hence, the whole trunk (thus, the tree) shifts in that direction. Think, what kind of coordination there should be in place so that the trunk remains stable and "looking up", not falling! And where is that "brain", which regulates this? So, our

belief that a brain is absolutely needed to "compute" and decide how to perform some clever action is wrong. This may be done without it. Making a choice of direction, in which to move, may be easier to explain. I read that, perhaps, the tree needs more sun. These trees live in tropical jungle of South America. At least I saw them there, and the direction it chose to move in was obvious. From what I saw, it looked like it was trying to escape falling into a deep ravine, perhaps, the result of the recent rain season.

The second kind of "thinking trees" we met in Cambodia, this was the so called giant Strangling Trees. These are huge, tremendously big trees, living around thousand years or more, which completely covered, fully destroyed the old cities of Cambodia. It was believed here that without any war the whole citadels and residencies of very powerful kings were suddenly left, and the nation went to build the new citadels somewhere else. These trees "eat" other trees. I witnessed this. The process starts with what looks likes not dangerous lianas use the trunks of the other trees for a support, and climb around them and up. When such a "liana" firmly establishes itself around a neighboring trunk, it starts joining with other "lianas" turning into one formation that becomes another trunk around the trunk of this tree's "dinner-tree". Gradually, it is all over the "dinner-tree", and it strangles and eats it. I think that at the previous stages of its growth it used to receive its food from the tree it invaded. I have photos of all the stages of this process. But these giant trees had also destroyed the buildings and huge structures in a more complicated way. I have difficulties to explain this in writing, though (see photos: on the top photo you can see another eaten tree).

If we started to accept this point of view on some other, then us, forms of life possessing consciousness and knowing how to "think" making conscious decisions, then having this view in mind we are ready to zoom out and widen the scale of living objects.

I'm returning to us, humans within the world of animals. It is very well known that we are not independent in our surviving needs. Our stomach is full of micro-organic living objects, our microflora. It is very much needed for digesting food, and in many other processes (say, creating some vitamins we need).

Also some of our own cells, like cells in blood, have semi-independent life. Our microflora does not know about our existence, it lives its own life and has its own consciousness. We may influence its existence by regulating our food, water and possibly some other supplies. We also "defend" it from changing conditions

outside our body. It lives in very stable conditions.   But it also may influence our life developing some sicknesses, or extracting some products, which may strongly influence our mood and behavior. We want to live in harmony with it, but we do not always understand how to do this (well, rather it is very seldom that we understand this).

Now, some curious "measurology" (I created this word joining "measuring" and "astrology", because, of course, the measuring I present does not prove anything, but, I hope, you will find it curious). Let's compare data on microbes from our microflora with respect to us, humans, being the place of their living, with the data on us with respect to Earth, our place of living. The size of individual typical microbe is smaller, than the size of a typical individual person, almost exactly the same as our size is smaller, than the size of Earth! So, we live on Earth with the same space as "they" live inside us. (One may start to worry about differences in the size of populations; however, there are billions of different types of living forms on the surface of Earth, as well as millions, if not more, types of microbes living inside us).

[Computation: the typical microbe is 1/10 size of our typical cell of the body which is around 1 micron = 10^{-6}) of a meter, one over a million, i.e., for a microbe it is 10^{-7}) of one meter. Our side is around 1,7 of a meter. So, by increasing it 10^7 times we will have 17.000 km. But the diameter of Earth is close to 13.000 km. The microflora of whales or elephants has even much more space inside their hosts for living, than these animals have on Earth.]

And now about living periods, the timing of our life. I mean how many generations of "our" microbes change inside us during our life. An average microbe divides every 20 to 30 minutes. Of course, for some it may take longer. And our own cells living much longer life; say, erythrocytes live around 90 days, but then they die, not divide. So, we have around 3 (or, say 2,5) generations per hour, and around 70 per day. Therefore, in one year inside of us there passed around 25.550 generations, and for 80 years (it is my 80-th anniversary now) we have around 2.044.000, roughly 2 million generations! To how many years of our presence on Earth this could be compared? Truly, this question doesn't make any sense from any point of view. However, I am interested in the psychological factor - the "feeling of time" of our species compare to what could be "reasonably" considered "the feeling of time" for microbes inside of us.

For instance, the notion of "generation" is different for us, than for microbes, who are "dying" (better to say, disappearing), when creating the next generation. While, if to consider for the length of time of one "generation" simply our life expectancy, i.e., this time around - 80 years, we could say that it would take 160 million years for humans to live on Earth for as many generations as our microflora changed inside of us. However, if a "generation period" is around 25 years (the expectation of age of the first child born), then the figure would be around 50 million years. Figures are relatively comparable. In any case, the life of our microflora inside of us by some objective parameters may be considered "comparably about the same", as our life on Earth, or at least, acceptably similar.

Why do I discuss this? To state that the life on Earth may have its own consciousness. Moreover, it should have it! Of course, it should be a huge intellectual power, so great that we cannot comprehend it from our very low intelligence. (Can a microbe inside us understand our existence?) This approach helps to answer many wildly open questions about life. But it also creates very many new questions. I engage now in briefly looking through some number of problems it would solve. And then I will list some questions it creates; some of them really worry me.

It seems obvious to me that even, if a microbe from our microflora would know that it lives inside highly intelligent body, it would be not able to establish a contact with us. At the same time, our micro/nano sciences are moving ahead very quickly, and, it looks like we would be able to establish contact with our microflora, and even with an individual microbe. The same, a human being and even all our humanity, I think, is unable to establish a contact with the super-intelligence of the life on Earth (assuming my conjecture is correct, and it exists). However, I think that superintelligence may establish such contacts. I, actually, think it observes us (as a whole) and helps. For example, we like to complain how unstable our weather became recently (actually, only very slightly unstable). As to me, I had been always on the opposite side, surprised regarding how our living conditions on Earth ARE STABLE.

Once in France (I forgot the year) suddenly from the North had broken winds, and they rushed through Paris and down to Mediterranean regions. This started with no warning, and went through with the speed above 200km per hour. Old huge trees had fallen down. People could not go out. Everything happened within a very short period of time, but forests and parks along the route of the winds suffered significant damage, often were completely destroyed. We tried to come to IHES in Bures-sur-Yvette - it stays in the park, and we could not enter. It was impossible to pass through the fallen trees. Never before and after the event I had encountered such a destruction; on top of this the whole damage was done in a very short time.   The

question is: how come this happened only once? Why not every year, many times per year? The difference in temperatures between, say, Norway and Algeria is 20-30 degrees C most of the summer. So, where are those terrible winds that should be expected acting in order to equal these temperatures?

I always have this feeling that weather is under some strange control. And not only the weather. Say, twice, when Russia was under deadly attacks and people were sure it will fall, in 1812 by Napoleon and 1941 by Hitler, Moscow (and Russia) was saved by enormously cold weather. It is usually much cooler in Moscow, than in Western Europe, but those years it was especially cold, unbelievable even for Russia.

Or consider the Gulf Stream in Atlantic Ocean. How such a miracle, the warm huge "river" inside the Ocean that moves tremendous amount of warm water from Mexican Gulf to the North of Europe started? Science may explain why it is stable, and why it exists. But not how it started. This happened around 15 - 20 thousand years ago and warmed the Europe. Ice fields retreated from Europe to provide the space for humans to develop and grow our intelligence.

So, I would think, the stable weather is one of the proofs of higher intelligence control.

And now the main point. How new species appear? How we, humans, appeared on Earth? Darwin!? Undoubtedly, natural selection and all the developed forms of Darwinism, do, indeed, influence development of species, "polish" and "perfect" their appearance and adjust them to the World conditions. But these processes don't create new forms, the completely different from those existing forms of living things, the new species! The probability of such creation through some "random events" is 0 (zero!); it is so small that trillions of years would not be enough to succeed, while our Universe exists only some (not quite established, yet) number of billions years. To state that it could happen randomly is the same kind of nonsense, as to state that randomly joining some blocks (even assuming such blocks do already exist) one may build a modern laptop. What an idiot would suggest this? But humans are much more sophisticated and very precisely acting creatures that accurately coordinate their activities.

So, the only possibility that comes to mind is that some higher intelligence created some (originally rough) copy of us, and then let the methods described in Darwinism to polish us and improve. (So, there is a room for Darwinism, too.)

Yes, it looks more like the standard religious "creation by God", than by the methods of "scientific Darwinism". However, I see creation standing infinitely far from being based on religious explanations. For example, I don't think that, if we pray, anyone will hear our praises and requests, and/or will pay any attention to them. I, actually, think that thousands years back our prophets, maybe, had understanding of this, and, as the outcome, was born genius idea of the single God. Yes, our Earth has a single life system, meaning a single superintelligence. But, as passing through the "middle ages", through the periods of our human intelligence falling, the only thing that was offering a hope of preserving intelligence was the great idea of a single super-intelligence that was primitively turned into the notion of God dwelling almost in human form (thus, bringing the better level of acceptance by primitive people).

I will call this single intelligence the "Creator", as it created us and the other living forms on Earth.

But NOT Earth itself! And not all other objects outside Earth, or, at least, outside its neighborhood.

And what about the life in other places of the Universe? For me this is a very easy question. I think, the same mechanism creates conscious systems on every suitably located planet around any star. And then this conscious (=living) system (=object) will create life of its own "microflora", i.e. living things on its territory. So, such living planets (or life on the planets) should be everywhere. We don't know about them similar to how our microflora does not know about other microflora.

Also, note, our microflora cannot feel that we, sometimes, try to help it. Indeed, remember the timing of life for microbes: around 20 min. It means that from the moment my doctor will tell me that I should help my microflora and take some specific tablets to the moment I, assuming to be a good patient and doing things quickly, take these tablets, a few days will anyway pass, i.e. 100 - 200 generations of microbes will be gone (!).

How similar should we expect it be, the life on other planets to our life on Earth? Well, how much differs life on different continents of our Earth? Say, how much differ souvenirs we buy in India and in Russia? They are different, not similar. However, serious stuff, like, say, cellphones, although produced in Korea, Japan or US, are quite similar.

Why is this important? Because I would expect these super intelligent systems on planets to be in contact with each other, so that to exchange important ideas. Souvenirs reflect taste and local understanding of beauty, and this may be circumstantially very different for different intelligence. But the essence of important discoveries is of common interest. So, animal life may be looking very differently, while the top achievements, although may slightly differ, are similar (compare Australian mammals, kangaroos and other

marsupials with North American and Euro Asian mammals). Top achievements of life, like human being, should be very similar. We are very well done!

A question that seriously bothers me in this schema is, why we, the humans, were created? And why now? The major difference between us and many other species is NOT a level of our intelligence. It is quite likely that some dolphins are not less clever, than us (or, perhaps, elephants?), and definitely many birds have a better built brain: it is much smaller, than the brain of us, humans, but it is known to do fantastic things and has fantastic active memories.

The main difference between us and any other life object on Earth is the ability to build, to construct. The language is also crucially important, but it on its own could be developed within application of the Darwin Theory methods of evolution, these would be enough.

But the combination:

i)   legs to walk

ii) hands to build

iii) the structures of "production of sounds" for developing of speech, and, of course,

iv)   high intelligence,

is unique and it is created only now!

So, why such species (as us) appeared only now?

Also scientific and technological developments are not uniform in time. After some very significant step made and, sometimes, even a real jump, we observe a long period of stagnation. To be able to absorb the jump.

What does surprise me in this? During the last 30 - 40 years the progress non-stop accelerates, while before that we observed the progress, to which there had been attached periods to absorb it, to get used to. But now we already have no time for absorption.

It looks like "someone" is in a hurry.

My questions now are: what for are we created? why such a hurry?

There are two directions of thinking along these lines. One is with worries regarding the danger to come. Perhaps, our Earth's life-system, this super-intelligence, needs our technological abilities, our help in saving ourselves and Earth's life.

The other direction is optimistic, and starts with the question: How is being created a new planet with a life-system on it ("living planet")? Here's one scenario. Some already existing life-systems may take care of reproduction that will create life on a suitable planet that is not, yet, made alive. Such planet may be located around some star on a not too big distance from, say, our Sun.

Let's for the sake of argument say the distance to such planet is below two light years from us. We may assume that reasonably soon there will be discovered the way of travelling in Cosmos with the speed of, say, 1/20 of the speed of light. Humans work on this right now. Then through us our Earth will be able to send to such planet its own life system, Earth's life system, that will arrive there in around 40 years. Presumably, some other living planets in the same neighborhood will do the same. Our cosmic ships with our messengers will meet on that planet and, mixing genes, there will be formed new life-system based on the life-systems of all the arriving ships. This way the new life-system will start on the targeted planet. Of course, in order to achieve this, the living creatures will need to work very hard for a very long time (long from our perspective). Many generations will pass before the planet will be filled with life of all arrived forms. The mixture of different genetics will help creating new stable forms, and gradually it will become the new alive system, new intelligence with its own interests and life. (And who cares now about such "small" detail as compatibility of different genetic, let me remind you that spermatozoids of one person come to a completely different person and are not being killed there; some well-known mechanisms stop non-compatibility effects; but discussion of this will take me to a few more pages of miracles).

So, perhaps, we, humans, will play role of "spermatozoids" on our living planet, on our Earth!

PS. To imagine the size of events, let us again compare "the birth of a new living planet" with the birth of a human child. It takes 270 days for a child to be born which means, perhaps, many hundreds or a thousand of generations (divisions) of cells, which in the end build our body. Similarly, from the start of the process of creating life on the new planet, many hundreds of generations of arriving astronauts should pass, i.e., tens of thousands of our years! Very slow (for us, humans) process, but very short for the life-system on the planet that is getting ready to exist for over half billion years.

7.   Conclusion

I glanced again through the text of this essay, and noticed one dramatic difference compare to my previous essay on similar occasion, "To-Day I am 70". The 10 years ago essay was full of different negative stories, more of them, than the positive ones. At some point in "To-Day I am 70", I had even explained the phenomenon; I have already mentioned this before, nevertheless I will quote myself: "...the number of episodes with a negative emotional content is greater, than the positive ones. We rarely retain positive feelings in our memory, unless they are absolutely exceptional. Thus, it's fresh air that we don't notice. Difficult, unjust events stay with us for a long time, sometimes forever. They weigh down upon our memory, and I have been almost glad to "get rid of them" by writing them down."

But in this text there are, perhaps, NO negative stories at all! This means I had "spilled them out" 10 years ago (I got rid of them!), and I had, indeed, changed since. Nothing is now too much painful, nothing of this kind stays in my head or stands in my way. It is a good, a very good feeling!

However, on the other hand, there are some situations that repeat themselves so often that I utilize a specific expression to describe them. I say: "For every good thing you did once you will pay later".

This sounds negative (and sad), but I also know that even, if I knew, that I were going to pay for that later, I would had, anyway, done the same good to all these people that I did during my life. This is called altruism, and this is, who I am. And I am happy with every case, in which I had helped during my life. Also, no doubt, there are enough people, regarding whom I am extremely happy that I helped them some time ago, and I never felt that I ever had to "pay" for that my help.

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Appendix 1. Advices for students (especially to mathematicians)

I collect here some of my advices for young mathematicians, which advices I've presented in "To-day I am 70". However, I also bring in some stories that illustrate our thinking patterns, demonstrate how our brain works, and I, also, suggest a few episodes about the working style of the couple of my friends, mostly of Bourgain, but also a little bit of Gromov.

About Brain work.

First, we should always remember that human brain is like a heavy train, and the better the brain is, the heavier is this train: if it (the train-brain) moves, it is not easy to stop it, but once stopped, it is almost impossible to get it in motion. This is why I do not allow my students to take too long a break: 2-3 weeks could be already too many. Misha Gromov once told me that the break from work of two weeks is the maximum he allows himself. (He was young back then. I think, later he would not allow himself even this.)

At the very beginning of my scientific career I experimented on myself in this direction. For example, I noticed that work of the duration of 10-12 hours straight, without breaks, from the evening till morning, would set my brain to a totally new state. Apparently, similarly to long distance running, a "second wind" comes, and your brain switches to a new mode. Those who have not tried it and have not experienced this state do not know the power of their brain, are not familiar with the powerful tool at their avail. I myself did not experience it too often, and by the age of 30 I was already physically unable to work in such an intensive mode. It is difficult for me to describe this state today, too many years have passed by, but I am still envious with respect to my younger self, who could experience it.

I often give my students the following comparison, to explain how our brain works. We see things precisely in front of us, like horses in the town, whose side vision is totally covered, so that they do not look around and are not afraid of what is happening to the right or the left of them. Any glancing to the side (off the beaten track) is tormenting, difficult, and for many, almost impossible. It is true that conservatism was important for our survival at early stages of human development. But now, we should try with all our might to be turning around, so that we could see the emerging of what is new. Only very few mathematicians do that often and well. It is a totally different parameter in our work, then an ability to prove even very difficult theorems.

Don't be afraid of "mad", "impossible" ideas. Check them. You may pass very close by a discovery and not see it. Here's a very short description of how things worked out, when I decided to write down some small improvement to the double-log estimate that I already knew. And all of a sudden, an insane idea came to my mind. A perfectly impossible one, so, at first, I attempted to shake it off. But then I decided to check it out, after all, and … everything began to come together, and the logarithm had disappeared! That was hard to believe, and it looked like in the Baron Munchausen story, where he was pulling himself by the hair out of a

swamp – I am talking about the special method of iterations, which I came up with (and which for some time was called Milman iterations).

This reminds me of one general principle, which one should always remember. In working on some non-trivial problem we sometimes stop, and cannot go through some point. It is like a wall, which stands ahead of us. Then, of course, we may try to go around it, or, if we did not succeed with this, we need to use our full mathematical strength and try to break it. However, this may also not work. But then remember, it may be a mirage, Fata Morgana, you imagine the wall, it does not exist - just try to go through it.

Let me mention now a couple of episodes involving Jean Bourgain, and his opinion on how we should be "cleaning" our brain to be able to achieve the maximal effect. It was 1984-1985 academic year, and we had just solved the isomorphic version of Malher conjecture. We quickly wrote a short CRAS article ("Sections Euclidiennes et volume des corps symétriques convexes dans R^n", 1985) and the question of writing the complete proof arose. My English was very poor, and, also, in general, I hated writing articles. So, I told this to Jean and suggested that he writes it: he had written so many articles and very quickly. His reaction was the following: "Vitali, I also hate to write articles. But one cannot think having hatred in his mind. So, to clean my mind I immediately write an article after I have proven the result: day time, if I have proven it in day time, or night time, if I have proven it at night!"

Yes, don't live with hatred in your mind! Keep your mind clean for Mathematics.

And another episode on this same subject.

In that academic year (84-85) Jean was very much worried about the fact that he writes too many papers, while he had collected already a very long list of publications (and he was just 30 back then). He said (and I agree) that people don't look inside the publications, and that most of experts will not expect every paper to contain a significant, leave alone very significant, result. They will mostly treat these as if they are just some remarks (and I do know mathematicians, for whom such a statement would be true). Consequently to this, Bourgain attempted to reduce the number of items on his list of research papers (those were times before "Google" or SciMathNet, so, it was our job to write and submit our own list of publications). We discussed this problem. It was already May and I said to Jean: " during this time, from October, you wrote around 6 or more papers on Maximal Functions" -and the last one, by the way, was, indeed, enormously difficult and important, and brought to him the position in IHES (I will talk about this later). "So"- I continue-" you could not publish all of them separately, but, then, you could place on your table all the ready text, and switch to the next one. Now, in May, combine them and publish this one absolutely superior paper that includes all the combined research". "NO" - reacted Jean. "This would be impossible. After a piece is finished and written down, I have to submit it to somewhere. Because even if it were typed and in every sense absolutely prepared, it would have, anyway, occupied some part of my brain, which will not allow me to be completely, 100%, involved in the next step. Submitting "frees" my mind, and I am ready to continue."

The same idea: free yourself for maximal intensity and concentration.

More, than once, I attempted to understand, how my brain works, how analogies come to mind, how an idea suddenly appears. It is hard to "catch" such thing. We immediately orient our attention towards the results, towards the end of that chain of thoughts, jumping from an episode to another one, and, even within a minute, trying to understand, how the thought came about, we face the fact that the whole transition has already "decayed", the brain has already forgotten about it. Just a few times I was able to grab that chain by its tail, and to unroll it in reverse, while it still has not disappeared from memory.

The results were amazing. Each time using a very clear analogy, the episodes were moving through 6-7 episodes from the starting one to the final result, but there was absolutely no connection between the middle links and either the beginning or the end. At the same time, the resulting final thought often had a perfect meaning; it was an important one. The intermediate links were not necessary! It may be obvious that at that time I was under the influence of James Joyce's so called "stream of conscience" notion (at that time for some reason his books, among others, were banned in Russia). By the way, I never "caught" a string longer than 7 links, and this, as I understood decades later, made sense. In the second volume of "Visions in Mathematics – Towards 2000" (GAFA 2000), I discussed it in the "Discussions at the Dead Sea" section, in its part about "Mathematics in Real World."

Consciousness.

To continue to discuss our thinking process, I will leave the brain power, and turn to something else.

 I will illustrate it through the strange feelings that has developed in me during the 84-85 academic year, closer to the spring. I started to feel getting closer to solving problems that I had been working on. My heart would race before I even knew the solution, and there was this strange feeling inside of me that somewhere in my subconsciousness it was all clear already, and now I just needed to "accept" it, and not let

it go. And, indeed, the solutions to the problems were coming to me. I think that during that year, every two weeks a new, non-trivial and often a well-known problem was solved.

To better illustrate what I am saying, I will deviate to describing our state of minds during and right after the First Gulf War with Iraq in 1991, when SCUDs that were launched by Saddam Hussain, Iraq's ruler, were falling on Israel. There is certain similarity of the above with how it felt, but the condition during the war is understandable, and is easier to describe. When a siren would go off, we would have about 90 seconds to prepare for the missile's impact. Everybody would jump out of their beds (this, usually, would happen at night) and each of us, including even children, would perform her/his task: turning off gas, electricity, hermetically sealing one room, in which we would all gather (we were preparing for chemical attacks), putting on the gas masks, cover our heads with a mattress (in case of a close hit, glass and other objects could fall on us from the walls and ceiling). Of course, a huge amount of adrenaline splashed into our bloodstream, but we didn't feel it, as adrenaline was working. Pretty soon our reflexes were so honed that a siren from an ambulance (or a police car) somewhere nearby during the day, which would be totally unrelated to an attack, would cause the same reaction and much of adrenalin produced. But it wasn't needed anymore, which we felt immediately. There was a very unpleasant reaction to the unused adrenalin: the heart was racing and everything inside was felt as if it fell and froze. This is a horrible sensation, and it takes time to get over it. To not keep Israeli citizens under constant pressure, the TV and radio stations during that period had been switching off the sound in their transmissions, whenever there had been happening in those the sounds of sirens. The ambulances tried not to use the sirens, either.

To continue with the previous story, the sensations I felt, when the solution to a problem would "leave" the subconsciousness and enter the consciousness, were similar. But instead of the sense of falling, there was a sense of a "lingering yearning" accompanied by rapid heartbeat.

It's possible that some other chemicals were released into the blood stream (or a smaller amount of the very same adrenaline), being responsible both for my condition and for the process of transition from the subconsciousness to the consciousness.

By the end of the summer I got scared. I was afraid that my heart wouldn't be able to handle it, but I could not stop the onset of those sensations. My wife, Ludmila, remembers how I began trying to convince myself that I no longer wanted to prove theorems, that I did not want to feel those sensations anymore, that I wanted a rest from them. And within a couple of months, they have, unfortunately, stopped. Several years later, when new mathematics of that year had been "digested," I tried very-very hard to stir up the same feelings within myself, to renew, as it were, my "contact" with the subconsciousness (which is a joke, but who knows), yet, nothing came of it. Only 20 years later, in the middle of the previous decade, several times I felt that I was very close to it, but no proof came out of that during those minutes, and the event was not completed. Thus, we always want what we do not possess, and when we have it, we are afraid of it.

Here is one more example of the connection between the subconsciousness and the body's "chemistry" involved. This example has to do with the most talented mathematician of our time, Ofer Gabber. First, a few words about him. Ofer was a 15-year-old university student in his last year, when I arrived to Israel. Then he continued at Harvard earning his Ph.D. and returned back home to Tel Aviv at the age of 18. By the age of 23 he was already a full professor of our department. I was back then the head of the Mathematics division, and managed to pull his promotion through the Senate, which was not trivial to do, since I was promoting such a young man; however, letters about him from the best specialists in algebraic geometry (the research direction, in which Ofer was working) were regarding him very high, which helped. Consequently, he became the youngest Math professor in the history of Israel. Yet, at the same time, he was an absolute perfectionist, a difficult predicament in itself, due to which he published almost none of his works (although they were all neatly written down and stacked on his shelves), and is, thus, known only within the circle of algebraists. But in that circle he is treated almost with a religious feeling. He often answers questions about problems, which had been evading the best minds for years, and does so "off the hip", during lectures, at seminars, and the whole of algebraic geometry has been moving ahead in the 1980s and 1990s under his influence. For example, expert number one in that science and those years, a Fields laureate Pierre Deligne, wrote in his letter to our University that during a conference he asked Gabber a question, which he (Pierre) had been working on for a whole year, and which he himself could not answer. A week of the conference has not yet passed by, when Gabber brought a full solution. Pierre added, "I was prone to thinking that I should quit mathematics, when I found out that the same thing had been happening with everyone around me."

In the beginning of the 1980s, I had been spending a lot of time on this back then, still, very young man, who was unlike anyone else I knew. It was commonly believed that I was the closest person to him, and

that only I could exert any influence on him. Stories about him could fill a book, but here I am only interested in one.

Deligne wrote a long article (about 200 pages), which was supposed to be a collaborative work with Gabber, who was supposed to read the text and give his conclusion and remarks. His perfectionism was delaying the publication of very important results, Deligne was getting nervous and asked for my help. I had a conversation with Ofer. His position was that there were mistakes in various parts of the work and that therefore he could not agree to its publication. "But it is impossible", I said, "that you would point out an error to Deligne, yet, he wouldn't correct it". "It is not that simple", Ofer answered. "They (this story has lasted for several years already, and the work has acquired new co-authors) want to present everything at such an abstract level, at which many details of the theory have never been properly verified and recorded. I can't point out specifically where is it, not all is going exactly as described, but, when I read an inexact or erroneous statement, I get a stomach ache, and, when I read this text, my stomach hurts all the time!"

I had nothing to respond to this. The work, which actually turned out to be a book about 350 pages long, came out without Gabber's co-authorship, although in the very first paragraph of the introduction it was written that the authors consider Gabber as one of the coauthors of this article, who, not being a mere mortal, could not take upon himself the burden of possible potential errors.

Thus, here we have a similar sign from the subconsciousness, a "stomach ache", or, more likely, unpleasant sensations inside the body. And, as I can judge from my own experiences, very unpleasant ones.

[I know of and witnessed more of similar Gabber's reactions, when without pointing to it he would claim: Stop! there is a mistake here. And he would be right. Once, in my presence, Gromov 3 times tried to explain to him some geometric point (Gabber was the algebraist, not a geometer), and was stopped by Gabber, who said that from some point he does not understand. It was the first meeting of Gromov with Gabber. It happened, when Gromov asked me to invite him to go to lunch together. As this conversation had been happening, I could observe Gromov getting angry, and could feel him thinking to himself: "what kind of an idiot have Vitali acquainted me with". But the third time around, Gromov had realized that he, Gromov, was wrong! He, also, could not see how to continue from that point. And he never objected Gabber from that point on].

Intuition.

Now, let me emphasize, that our thinking and comprehension are based on intuition, and only, as presenting our line of thoughts, we write them in a formal way. Very often, intuition is taken out of formal contexts, and with this the research articles often impoverish themselves. To get an "unexpected", surprising result gives the greatest pleasure, and indicates a change of intuition. That's because "surprising" means not corresponding to established intuition, not continuing the way we were accustomed to think. The way I see it, a change of intuition defines the arrival of a new direction in science, in Math, particularly.

In the mid 80-th one result after another in our direction of Mathematics demonstrated the destruction of old intuition, built a new intuition and, likewise, the new field of Mathematics, Asymptotic Geometric Analysis. Yet, I don't know how to define the intuition, I don't know what is it? But I know that students should be spending a few years intensively working on one subject to develop certain amount of intuition. There is no short-cuts here; some time is necessary. Without this no Math problems would come to your mind. You will not distinguish an important, an interesting question from what ancient Greeks characterized by saying that one foolish man may ask a question, to which the hundreds of clever ones would not find an answer (and I would add to this that often such questions do not need to be answered.)

Style of working.

There is no the unique style to do mathematics extremely successfully.

 Gromov, say, needs absolute quietness. It is close to a mad point for him. He may stay in a place which looks to me absolutely quiet, and, anyway, put on his ears the best headphones, not to hear music but to close himself additionally against any sound. This leads, sometimes, to very funny situation. Once we agreed to meet in his apartment in Paris. I came at agreed time, rang the bell, did it many times, and then left. I knew he was inside, but just worked and did not hear. But I did not leave for good, no, I went to the closest public phone (cell phones were not, yet, in use), and called him. Only after the third ring, he took the phone (perhaps, he actually saw that it rings), was very apologetic, and rushed to open the door.

But for Bourgain noise was never crucial. Jean mostly worked during the first part of the night. This was his way to be isolated, undisturbed, having absolute quietness. However, from time to time he also needed an atmosphere of "white noise". For that he often walked along the Champs-Elysees. Sometimes, for the same goal, he would had taken a metro from Bures-sur-Yvette to Paris (about 40 minutes), and would had immediately returned. By the way, his walks along the Champs-Elysees led him to relatively funny episodes,

for any professional "pockets master" does immediately distinguish a person, who is deep in his thoughts, is completely inside himself, and such "master" would, of course, try to use it.

On the same topic, I was once told the story about Gelfand. He was asked by some foreign visitor (it was in Moscow), how one should do mathematics (no Russian-based mathematician would have asked him this). So, Gelfand answered: "if you want to write many many papers, wake up at 7 am and work all the time. But, if you want to discover/prove only a few, but outstanding results, then don't bother coming anywhere before noon". (This isn't just a joke. Many of my discoveries were made during such morning hours, when I wasn't in any hurry, and could stay late in my bed.)

Style of a presentation.

I've already touched this subject in my above "Intuition" piece.

It is a difficult task for a young starting his scientific career fellow to write his/her first paper. I heard a story from Arnold that Kolmogorov (who was the adviser of Arnold) would usually himself write his student's first paper. Indeed, if someone gets her/his first Math results, and Kolmogorov presents her/him with these results written down for publication, this, perhaps, is the finest way to explain, how the writing should be done.

My experience was different. My advisor, B.Ya. Levin asked N.Akhiezer to help me with the writing. N.I.Akhiezer was a great master of writing. It was always a pleasure reading his articles and books. So, he took my written text, and started in my presence changing it. The work was meant to become a short note to "Doclady Academii Nauk" of USSR, a Russian version of CRAS, it didn't contain proofs, only the results and some ideas. I remember one of his general advices: "Vitali" - he said - "the paper should be like a candy ("konfetka", in Russian), it should be a pleasure reading it, just like a feeling of a candy in your mouth".

Since I am not sure that know, how to well write Math articles, my advice to my students is the following: go to library (to computer these days) and read different articles on the same subject. But not necessarily with the goal of understanding them. Just in order to see which author writes in "your style", so that it feels easy and pleasant read to you. Then read a lot of text of this author, and, thus, adapt his/her style - write similarly. We are all different people, with our own specifics of how our brains work, hence, we are likely to have different styles that would suit us. So, find someone, whose style suits you, and start copying. Later you will bring in your own style.

I will add that one should decide, what is her/his expected audience for the paper. Of course, my advices here are under assumption that the paper contains some very significant advance. Then, if you write it for the experts in your own field, and you happened to solve some problems that these people attempted to solve and failed, and on top of this, if they trust that you, indeed, solved it, then you may not care, whether they will understand your text, or they will not. If their level in this field is low, they would even respect you more. However, if the audience of the top class, and such that it may be expected to be not only from your own field will not understand you, they will consider you to be an idiot, not themselves! So, if you want to impress the top crowd, you better write your paper so that they could understand it. This is at your highest interest to attract their attention to you and your Math.

One more very important remark. Avoid introducing "pollution" to Mathematics! In general, pollution in mathematics refers to unnecessary or poor definitions and concepts. Definitions, just like theorems, should be well thought through. Definitions that do not fit in with the purposes and concept clutter the mathematics and can no longer be used, where they would be more relevant later. Not quite successful mathematicians often replace bold results, which they don't have, with the bold definitions that keep little behind them.

In connection with this I remember a funny episode. In Israel, in 1997, due to receiving his Wolf prize, physicist-astronomer, John Wheeler, gave a lecture. It was given in a full to overflowing very large auditorium of our faculty. So, this is what I remember that he said about the role of words in science. "Don't underestimate the importance of the well-chosen word in science," he said – "if I hadn't thought of the expression 'black hole", I would not have become famous, nobody would know anything about this field, I would not have received the Wolf prize, and would not be standing here now." Everyone laughed, but there was profound truth in that. Ill-chosen vocabulary and poor delivery may put people off the field, and this, certainly, does not help to attract them.

By the way, search for pollution inside your own text, and this way oriented analysis of what are you doing, may bring absolutely unexpected discoveries.

Here are some examples.

One of the most important concepts (constructions) in geometry (and also in mathematics in general) is the concept of polarity. In Functional Analysis, its analogue is the concept of duality, which is also known as conjugacy or adjointness. These are absolutely fundamental concepts and non-trivial constructions lead to

them. The question of how mathematics came to these constructions, the story of their creation, often interested me in the past. However, I never (in the past) asked myself, what one should call duality. We considered the complex constructions that led to it, for something God-given and natural. In one of our works together with Artstein and Klartag, we had expanded the concept of polar, and defined it for a special, widely used class of functions, i.e., not just for sets. This would have been fine, but soon the reasons, why we had given name "polarity" to the operation that we had carried out, began to cave in. And it turned out that our concept could be rather pollution, than a necessary and natural definition. And the question arose as to what is polarity, and whether our definition and concept were justified. In a little while, we (Shiri and me) understood everything, (and yes, the concept we had introduced turned out to be the only one possible!). We did not introduce pollution to Mathematics!

And the picture, which had opened up to us, turned out to be surprising and unexpected. It was very much worth to clarify the notion and check it on a possibility to be a pollution. If we would have not cared about this, we would had missed the enormously beautiful and, also, important piece of mathematics.

Polarity (which we also call duality) turned out to be an operation, reversing the direction of inequality - if we are dealing with functions, or the direction of embedding - if our object consists of sets. Of course, the class of functions (or sets), in which we study duality, plays a central role, and in different classes different (and non-trivial) formulae arise for describing the operations of duality. In the simplest case of a linear class of real-valued functions, it is simply the minus sign, but for the class of non-negative functions it is inversion, i.e., the operation that maps f to l/f. However, for a more interesting class of all convex functions, it is already the Legendre transform (and only that; what makes the interest of our results is that we describe all the transforms that change the sign of inequality for functions). [Of course, those, who are interested in these Math topics, should turn to published works for the precise definitions and formulae.]

The described story had many more consequences, than it may seem it could have. For one, it changed my view on Mathematics, and I spent next 10 years mostly digesting it. In this research, what surprised me right away was how little one needs from an operation for its unique restoration. For the latter, the elementary conditions (changing the sign of inequality) call forth the appearance of formulae, which are not at all obvious, different formulae for different classes. Lately, we (Shiri and I) have discovered a class of functions, for which there exist exactly two (different) types of transforms reversing the direction of inequality, and one of these types turned out to be a new transformation, a new duality, not known before. The role of Shiri in this discovery was central.

To me it was a certain shock. Mathematics turned out to be much poorer, than I was accustomed to think (there is a positive way of saying this, as well: mathematics is stable – "rigid"): the most interesting and significant transformations uniquely arise out of the simplest and elementary conditions.

I immediately decided to test this for the Fourier transform. And really, we showed, (first together with Alesker and Artstein-Avidan, and later, more sophisticated and simpler formulated facts, jointly also with Faifman), that the Fourier transform uniquely arises out of the conditions of the exchange of the operations of multiplication and convolution (and, of course, again, it is better to read about this in our published work). Now we know many additional similar examples which I will mention later.

Search for discoveries.

Let's start with an example.

More, than a hundred years ago, in 1888, certain inequality had been discovered by Brunn and later developed by Minkowski; consequently, for the last hundred years it carried the name of Brunn-Minkowski's inequality. This inequality is easy to prove (I know a dozen of its proofs), yet, it possesses an incredible strength, and is one of the most important geometric inequalities, the one that had created modern theory of convexity. Amongst its immediate consequences, for example, is isoperimetric inequality in linear spaces. The latter had been occupying the minds of mathematicians since ancient times. In my view, much of real important research had been performed in the centuries preceding this discovery. Thus, such mathematicians as Euler, Gauss and many others overlooked the jewel. Was this an accident? We go ahead along this road called mathematics, the jewel like the inequality of Brunn-Minkowski lies there, but we pass by and don't notice it.

Perhaps this is an accident, but are there no other such slips?

Actually, I think the opposite. In recent years we discovered a few more of such oversights. And I have already described one of them in my previous piece here. I will quickly go through the list. Our result with Shiri Artstein about the Legendre transform ("The concept of duality in convex analysis, and the characterization of the Legendre transform". Ann. of Math.(2) 169 (2009), no. 2, 661–674.) should have been known already for at least 150 years, but it was not. The same about our result with Shiri and Hermann Koenig

about characterization of the derivative ("The chain rule as a functional equation." J. Funct. Anal. 259 (2010), no. 11, 2999–3024) or the characterization of the Fourier transform (a few papers Alesker, S.; Artstein-Avidan, S.; Faifman, D.; Milman, V. "A characterization of product preserving maps with applications to a characterization of the Fourier transform." Illinois J. Math. 54 (2010), no. 3, 1115–1132 (2012); Artstein-Avidan, S.; Faifman, D.; Milman, V. "On multiplicative maps of continuous and smooth functions". Geometric aspects of functional analysis, 35–59, Lecture Notes in Math., 2050, Springer, Heidelberg, 2012).

Also a series of our joint results with Hermann Koenig (see the book "Operator relations characterizing derivatives. Birkhäuser/Springer, Cham, 2018. vi+191 pp.), which followed the above results, has such classical spirit and had to be known for decades, if not centuries. I think today that our whole road of mathematics is strewn with jewels, which we do not notice. Of course, one should notice them, and clean them, and find the right frame for them, and, in any case, one should remember, should believe that they are everywhere around us, and, especially, I address this to the young mathematicians.

Hermann Koenig and Vitali Milman, around 1985

A few general remarks not aiming any specific goal.

Usually, in our memories the number of episodes with negative emotional content is greater, than the positive ones. We rarely retain positive feelings in our memory, unless they are absolutely exceptional. Thus, it is fresh air that we don't notice. But difficult, unjust events stay with us for a long time, sometimes, forever. They weigh our memory down, and it is good to "get rid of them" by, say, writing them down.

I would like to note one surprising, but natural, thing: excellent organizational skills and an understanding of situations and people that someone possesses very negatively affect recognition of such a person as a high-level scientist. Of course, the gift of a good organizer is as rare as the gift of a high-level mathematician. These are "independent" abilities (we may say "independent events"), so it is very rare for one person to have a combination of these skills. This is why a majority of good organizers in science are not high-level scientists. This organizational gift is instantly obvious. There is no need in analysing any non-trivial scientific work in order to admit that someone has this gift. Then this is just a reflex to question the academic level of a scientist, who is clearly successful in his organizational efforts.

After arriving to the West I had quickly realized that this World did not recognize universalists. Even within mathematics itself one first had to become an absolute expert in one particular field. Only an already recognized expert "had the right" to earn additional bonus points doing work in other areas of mathematics, but working in the fields of, say, biology or medicine was left only to the absolutely recognized world-class experts in mathematics. (However, by now there have appeared new areas, such as bio-mathematics, which sit at the crossroads of sciences, and, therefore, these areas have their own experts, who do not "come from the outside.")

Appendix 2. " Who I am" and what is "my mathematics" from outsiders perspective.

I wrote in the beginning of this essay that the question "who I am" is irrelevant to me by now, and the reason for this is that I know the answer.

[By the way, the last year on the occasion of the 70-th anniversary of the State of Israel, the Milner Global Foundation jointly with Ha'aretz-Themarker Newspaper selected 70 scholars (I cite now:) "who are/were Israeli citizens who have made groundbreaking contributions in their respective fields". The list includes, also, Israeli citizens not living in Israel, and, also, those, who had already passed away; it goes across all directions of Science. I am one of the selected on this list, and consider this to be the highest honor I ever had. The English version may be seen:

However, some photos in the English translation are wrong; so, see also Hebrew version:
]

However, it is curious to know the details of how people see me from outside, especially experts, and even more, how they estimate my field of Mathematics, in which I was involved a few last decades. By the way, their positive opinions on myself are, sometimes, I think, exaggerated. Reading some of these opinions, I, sometimes, catch myself on being quite a modest human being. For example, in one report on my grant I read:

"Vitali Milman is a force of nature who has almost single-handedly created the area of high-dimensional convexity theory over the last 30 years. It is now one of the most vibrant areas of analysis attracting huge interest."

Of course, these opinions on submitted grants are sent to us, the proposers, in anonymous form by the authorities of the Grants; this means that I don't know, who wrote them, but grants are sent for reports only to highly qualified experts. Also, they are sent to us by a usual mail, not e-mail, and separately from the grants. So, it is problematic recognizing now, many years later, the dates of when the particular opinions were written. (In my estimation the above cited was written around 2010.)

Below, I will show off with a couple excerpts from reports on my different grants. Usually, we received 5 to 6 such reports on every grant (and, usually, we submit two different grants at the same time for the duration of 3 to 4 years). Therefore, of course, we have a lot of such reports, but in my case they are all in the same general spirit, although written by different people, which, actually, must be proving that they represent a consensus.

I must say that only now I paid attention to this side of advantage in writing Proposals: they allowed to the Proposer to see what people write about him/her, what do they think about her/him and his mathematics.

"The scientific program of Vitali Milman has been one of the great success stories of Analysis in the last quarter century. He has created new direction (Asymptotic Analysis) and deeply influenced others (convexity theory, Probability theory). Overall the impact of what he did was enormous. The most remarkable feature of the work of Milman is that he has been able to see much before others where the fruitful directions were going to be. He is obviously still bringing with ideas, some of which might turn as important as past ones."

Well, since I don't know when this was written, I can't estimate, whether the author of this report could be expected to get disappointed later.

I would like now to cite the opinion of different experts in the field of Mathematics I was involved, And I will omit all flattering words about my role in the story. I start:

"Asymptotic geometric functional analysis has had a tremendous impact on many areas of mathematics and other sciences and has revolutionized the classical field of convexity."... " The main lines of research and the techniques for developing them are by now well developed (and in the continuation of the report)...this is developed like "the extraordinarily powerful machinery of the area (which, by the way, is mastered by only a few researchers)".

There is one point in this report, on which I would like to comment. It is written: "He has, after all, for almost 40 years been one of the most creative thinkers in a broad range of analysis and shows no signs of slowing down."

And my comment is (I am trying to be funny): This must had been written, when there had been at large expectation of me "slowing down" :). Many of my colleagues complained about it: why are you not slowing down? I did - now. I feel I slowed down very significantly, almost to zero. Haven't I made many people happy? :).

"The concentration phenomenon (pioneered by the proposer) has had a number of striking consequences in this theory, most especially the reverse Brunn-Minkowski inequality and the Q.S. (=Question of Subspace) theorem. More recent the work of Talagrand and others has shown that related ideas are extremely important in statistical mechanics. ... . There is no doubt that the field of research is important in its own right and has implications in a variety of other crucial areas of mathematics".

"A very important idea in the project is that it recognizes a deep connection between high dimensional convex geometry and asymptotic results in discrete mathematics and computer science, and also emphasizes the connections with concentration phenomenon in probability and physics. .... It recognized high dimensionality as an entirely new discipline, and initiated broad research in this area..."

And from a very similar report I cite only the opinion about the field:

"...a new branch of mathematics: Asymptotic Geometric Analysis which combines of two different theories, namely classical convexity theory and the theory of finite dimensional normed spaces highly successfully to get deeper insights into both and also other branches of mathematics."

"The emergence of this new field of isomorphic asymptotic theory of convex bodies, .., was a totally new and unexpected development to practitioners of classical convex geometry, … ."

(And so on.)

Now, why won't I add to the above list of the anonymous opinions a few more that were either published or sent to me directly, i.e. the signed opinions.

Misha Gromov wrote to me in 1996:

"Right now I am reading about Milman ellipsoid, it is a marvel, m."

Of course, the main fascination of Gromov (in what I brought to Mathematics) is with Concentration Phenomenon, which he, actually, calls "Levy-Milman concentration phenomenon" and writes in his book [Gr1999], Chapter 3 and a 1/2:

"Levy's fascinating results and ideas had remained largely unknown for 20 years (in my computation - 50 years - V.M.; Misha saw only the second edition of Levy's book which is originated in the first edition from 1922, and which is, actually, the publication of Levy lectures in 1919; but whatever we discussed now, was written already in the 1922 book - V.M.) until Vitali Milman realized their importance and ubiquity. .... Milman extended Levy's result... and then pushed forward the idea of concentration as a general unifying principle. Ever since, Milman vigorously promoted the concentration phenomenon, as he called it, and the idea was reluctantly accepted by the community of functional analysts."

This idea of Concentration is standardly presented in the setting of mm-spaces: metric-probability spaces. However, the idea may be adapted to the metric spaces only (we did it with Gromov, but did not publish). Later Pestov picked up this, and one may also see it in our joint book with Artstein-Avidan and Giannopoulos (Asymptotic Geometric Analysis, Part 1); this concentration idea may be also adapted to probability spaces without metric structure. We did it with Giannopoulos, and again, some note about it may be found in the book I have just mentioned.

Since this piece, anyway, turned out to be a discussion of my role in the Concentration Idea I will bring two more opinions from Talagrand and Gowers.

Talagrand's opinion is of a special interest, as he developed and move this method very far. So, from Talaghrand (1995, March)

" Dear Vitali,

You do, indeed, have good taste. Concentration of measure was a good choice of subject. I have been chosen as the main speaker for the 4th world congress of the Bernoulli society, in Vienna in 96; expected attendance > 1000...Three hours of plenary lectures with discussion of the lectures afterwards...

Michel   "

There is a story behind this message, which explains it.

Michel Talagrand is enormously productive and strong mathematician. He solved during 80-th many very different problems of Analysis and Probability, and worked very hard. However, a recognition was not coming to him. Perhaps, the interest to directions he was involved in was low. So, once he came to Pisier, and asked for his advice, asked what he should further do, what problems he should work on. The advice of Pisier was: "Go after Milman, his talks and his research, and you will find what to do". And Michel did it. This story Talagrand told me himself. He called me "his lucky chance". And indeed, I published my last paper-survey on Concentration Phenomenon in 1987 (dedicated to the memory of Paul Levy; it was a huge his 100 years commemoration conference), and Michel picked up the subject from that point on. His first very long (and excellent) paper on the subject was published by IHES –Publications (M.Talagrand, Concentration of measure and isoperimetric inequalities in product spaces. Inst. Hautes Études Sci. Publ. Math. No. 81 (1995), 73–205. Very respected journal, by some measuring it is the World #1), and it is written there that the paper is devoted to me. This, I guess, explains the quotation above.

Also, in his another paper in "Annals of Probability" in 1996 Talagrand writes: " The idea of concentration of measure (which was discovered by V.Milman) is arguably one of the great ideas of analysis in our times. .... This concept now plays an important role in the local theory of Banach spaces and the dominant role in probability in Banach space. (This author is, in particular, pleased to acknowledge that his contributions in this direction have their ultimate source in Milman's philosophy.)"

(Similar statements are written in many other papers on the use and development of Concentration Phenomenon; See, e.g. S.Bobkov , M.Ledoux, and others).

And the promised piece from T.Gowers article in "Mathematics: Frontiers and Perspectives",AMS,2000 :

"The full significance of measure concentration was first realized by Vitali Milman in his revolutionary proof [Mil1971] of the theorem of Dvoretzky. ..... . Dvoretzky's theorem, especially as proved by Milman, is a milestone in the local (that is, finite-dimensional) theory of Banach space. While I feel sorry for a mathematician who cannot see its intrinsic appeal, this appeal on its own does not explain the enormous influence that the proof has had, well beyond Banach space theory, as a result of planting the idea of measure concentration in the minds of many mathematicians. Huge numbers of papers have now been published exploiting this idea or giving new techniques for showing that it holds."

……………………….

I will finish this article with the following funny piece; it looks like a joke. This is what I received in 1999 from Haim Wolfson, who was my PhD student in the end of 70-th start of 80-th (later, around 10 years ago, he was the Dean of the Faculty of Exact Sciences):

" Shalom Vitali,

I want to share with you a funny episode I had with my spell checker of Netscape mail. I wrote an e-mail letter which mentions Weizmann Institute. It appears that the name Weizmann was not in the dictionary of the spell checker, so it suggested me to correct this word. Usually, the program suggests a list of corrections. In this case there was only one word on the list that the program decided that it is close enough to "Weizmann". It was "Milman". What do you say about it?

Best Regards, Haim "

I am infinitely thankful to my brother Vladimir Milman who has polished my poor English. Of course, if I later add some new parts, the reader will immediately recognize them, because of them being written in my original English.