Martin and Floyd boundaries of finitely generated groups

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Abstract. The talk is based on two recent preprints

1. [GGPY], I. Gekhtman, V. Gerasimov, L.P. W. Yang, *"Martin boundary covers Floyd boundary"* (arXiv:1708.02133),

2. [DGGP], M. Dussaule, I. Gekhtman, V. Gerasimov, L.P. *The Martin boundary of relatively hyperbolic groups with virtually abelian parabolic subgroups*" (arXiv:1711.11307).

We study two different compactifications of finitely generated groups. One is the Martin compactification which comes from the random walks on the Cayley graph of a group equipped with a symmetric probability measure whose support generates the group. It is the closure of the set of harmonic functions on the group in the set of all real functions endowed with the topology of pointwise convergence. The second compactification is the Floyd compactification which is the Cauchy completion of the Cayley graph equipped with a distance obtained by rescaling of the word distance by a decreasing function. The corresponding boundaries are the remainders of the group in these compactifications, they play important roles in the analytic and geometric group theory.

Our first main result from [GGPY] states that the identity map on the group extends to an equivariant and continuous map between Martin and Floyd compactifications. We also prove that outside of a countable subset of the Floyd boundary preimage of a point by this map is a point. The proofs of these results are based on our generalization of the Ancona inequality proved by A. Ancona for hyperbolic groups in 80's.

Using these results we describe in [DGGP] the Martin compactification of a relatively hyperbolic groups whose maximal parabolic subgroups are virtually abelian. In case of a nonuniform lattice acting on the real hyperbolic space, it is obtained as the closure of the space with a maximal invariant system of disjoint horoballs at parabolic fixed points removed. A topological description of the Martin boundary of relatively hyperbolic groups with respect to virtually nilpotent subgroups remains a largely open and intriguing question.