

Title: A few fairy Math tales

Abstract:

The format of this talk is rather non-standard. It is actually a combination of several mini-talks. They would include only motivations, formulations, basic ideas of proof if feasible, and open problems. No technicalities. Each topic would be enough for 2+ lectures. However I know the hard way that in diverse audience, after 1/3 of allocated time 2/3 of people fall asleep or start playing with their tablets. I hope to switch to new topics at approximately right times. I include more topics that I plan to cover for I would be happy to discuss others after the talk or by email/skype. I may make short announcements on these extra topics. The topics will probably be chosen from this list:

“A survival guide for feeble fish”. How fish can get from A to B in turbulent waters which maybe much faster than the locomotive speed of the fish provided that there is no large-scale drift of the water flow. This is related to homogenization of G-equation which is believed to govern many combustion processes. Based on a joint work with S. Ivanov and A. Novikov.

“What is inside?” Imagine a body with some intrinsic structure, which, as usual, can be thought of as a metric. One knows distances between boundary points (say, by sending waves and measuring how long it takes them to reach specific points on the boundary). One may think of medical imaging or geophysics. This topic is related to the one on minimal fillings, the next one. Joint work with S. Ivanov.

One of the greatest achievements in dynamics of the XX century is the KAM Theory. It says that a small perturbation of a non-degenerate completely integrable system still has an overwhelming measure of invariant tori with quasi-periodic dynamics. What happens outside KAM tori has been remaining a great mystery. The main quantitative invariants so far are entropies. It is easy, by modern standards, to show that topological entropy can be positive. It lives, however, on a zero measure set. We were able to show that metric entropy can become infinite too, under arbitrarily small C^∞ perturbations. Furthermore, a slightly modified construction resolves another long-standing problem of the existence of entropy non-expansive systems. These modified examples do generate positive metric entropy is generated in arbitrarily small tubular neighborhood of one trajectory. The technology is based on a metric theory of “dual lens maps” developed by Ivanov and myself, which grew from the “what is inside” topic.

How can one discretize elliptic equations without using finite elements, triangulations and such? On manifolds and even “nice” mm-spaces. A notion of ρ -Laplacian and its stability. Joint with S. Ivanov and Kurylev.

“Area spaces”. Given a Riemannian manifold, one can look at rectifiable 1-cycles and their minimal fillings by 2-cycles. Actually, one looks at the completion of this Abelian group with a norm. All information about concrete points is gone, this is just an abstract object, more or less an infinite dimensional space with a lattice. What can we recover back about the manifold? It turns out that, in dimensions bigger

than two, just everything: both the topology and Riemannian metric. On a joint work with S. Ivanov, with contributions by C. Sormani and N. Higson.

Ellipticity of surface area in normed space. An array of problems which go back to Busemann. They include minimality of linear subspaces in normed spaces and constructing surfaces with prescribed weighted image under the Gauss map. I will try to give a report of recent developments, in a nutshell. Our interest to these problems came from our attempts to attack questions discussed in "what is inside?" mini-talk. Joint with S. Ivanov.

How notions of Bi-Lipschitz equivalence and quasi-isomorphisms are related? On the language of separated nets, the problem comes from Gromov and Furstenberg (from different angles). What does this have to do with the following question (asked by Moser): Is every positive continuous function bounded away from zero and infinity is the Jacobean Determinant of a Lipschitz homeomorphism? What is the relation between these problems? On a joint work with B. Kleiner.

Are there partially hyperbolic diffeos of the three-sphere? More generally, of three-manifolds with Abelian fundamental groups or even groups of sub-exponential growth? There are no topological obstructions. Still, the answer is "no" The result uses a rather complicated study of co-dim 2 foliations. Joint with (partially) M. Brin, and S. Ivanov.

A seemingly naive question. Which groups admit bi-invariant metrics of infinite diameter? For unknown reasons, this problem was overlooked. It turned to be not at all easy. Of course, infinite Abelian groups admit such metrics (though it requires a proof). Groups with non-trivial quasi-morphisms (such as hyperbolic groups) admit them too. For groups of geometric origin, the story becomes really intriguing. Joint with S. Ivanov and L. Polterovich.

Quite a few stories are left in my left pocket.