Titles and Abstracts FDIS 2022

1. Andreas Knauf:

   Title: Nondeterministic dynamics of point particles.
   Abstract:
   We consider the motion of n particles in d dimensions. Between collisions they move with constant velocities. At collision the center of mass moves with constant velocity, too, but the total kinetic energy is not necessarily conserved. So there can be infinitely many collisions, which is the case we are most interested in. We compare this nondeterministic dynamical system with the temporal asymptotics of deterministic particles, in particular the n-body problem of celestial mechanics.
   This is joint work with Manuel Quaschner (Erlangen).

2. Ivan Izmestiev

   Title:Darboux's wreath of 12 surfaces

   Abstract: Finding an isometric deformation (a bending) of a surface is a classical problem in differential geometry. A linearization of this problem leads to the notion of an infinitesimal isometric deformation (IID). An IID is a vector field along the surface such that deforming the surface along this field preserves the lengths of all curves in the first order. A surface and its IID can be both seen as smooth maps from an abstract smooth surface to the space. To a pair (p,q) of a surface and its IID one can associate a further pair (r,t) of smooth maps, called the rotation and the translation fields. It turns out that t is an IID of r. Thus the construction can be iterated. A remarkable fact discovered by Darboux is that this dynamics is periodic: after the sixth iteration one recovers the initial pair (p,q). The twelve surfaces in these six pairs are related in several other ways, which led to calling them the Darboux's wreath. In this talk we will present a recent new proof of the periodicity found by Bruno Sevennec and based on triality, a set of relations between the points of the projective quadric with signature (4,4) and the two families of maximal linear subspaces in this quadric.

3.Alexey Bolsinov.

Title: Nijenhuis geometry, Poisson brackets of hydrodynamic type and intergable multicomponent PDEs

Abstract:The talk is a survey of recent results on applications of Nijenhuis geometry in the theory of integrable PDEs.

4. Alberto Abbondandolo

Title: A normal form for perturbations of periodic Reeb flows

Abstract: I will discuss a normal form for contact forms that are close to one whose Reeb flow is periodic, together with its applications in Riemannian and symplectic geometry.

5. Sasha Veselov

Title: Integrability in Thurston's geometries and knots

Abstract. I will review the situation with the integrability of the geodesic flow on the three-folds admitting one of the 3 group geometries in Thurston's sense. A concrete case of the modular 3-fold SL(2,R)/SL(2,Z), known to be homeomorphic to the complement of a trefoil knot in 3-sphere, will be discussed in more detail in relation with the remarkable Ghys results, linking modular knots with periodic orbits in the celebrated Lorenz system studied by Birman and Williams. It will be shown that in the integrable limit of the corresponding geodesic system we have the isotopy class of cable knots of trefoil. The talk is based on a joint work with A. Bolsinov and Y. Ye

6. Rick Kenyon

Title:"Dimers and SL_n".

Abstract: We consider SL_n-local systems on graphs on surfaces and show how the associated Kasteleyn matrix can be used to compute probabilities of various topological events involving the overlay of n independent dimer covers (or "n-webs"). This is joint work with Dan Douglas and Haolin Shi.

7. Anton Izosimov

Title: Polygon recutting

Abstract: Recutting is an operation on planar polygons defined by cutting a polygon along a diagonal to remove a triangle, and then reattaching the triangle along the same diagonal but with opposite orientation (V. Adler, 1993). Recuttings along different diagonals generate an action of the affine symmetric group on the space of polygons. I will show that this action is given by cluster transformations and is completely integrable. The integrability proof is based on the interpretation of recutting as refactorization of quaternionic polynomials.

8. Eva Miranda

Title: To b or not to b, that is the question

Abstract: Motivated by the study of the index theorem of manifolds with boundary, Melrose introduced b-calculus to work on manifolds with boundary. Later on, Nest and Tsygan used it to investigate deformation quantization of symplectic manifolds with boundary. Indeed b-structures and other generalizations (E-symplectic structures, folded symplectic structures, etc...) are ubiquitous and they are hidden, unexpectedly, in many other places including the space of pseudo-Riemannian geodesics (Khesin-Tabachnikov), in regularization transformations in celestial mechanics (Mc Gehee) and in Yang-Mills theory.

This talk will provide a panorama tour of the theory focussing on open problems in Hamiltonian Dynamics such as the Weinstein and Arnold conjecture. The study of integrable systems, periodic orbits and reduction theory will be some of the main characters of this talk.

9. Alexey Glutsyuk

Title: On rationally integrable planar dual and projective billiards

Abstract: A caustic of a strictly convex planar bounded billiard is a smooth curve whose tangent lines are reflected from the billiard boundary to its tangent lines. The famous Birkhoff Conjecture states that if the billiard boundary has an inner neighborhood foliated by closed caustics, then the billiard is an ellipse. It was studied by many mathematicians, including H.Poritsky, M.Bialy, S.Bolotin, A.Mironov, V.Kaloshin, A.Sorrentino and others. We study its following generalized dual version stated by S.Tabachnikov. Consider a closed smooth strictly convex curve $\gamma \subset RP2$ equipped with a dual billiard structure: a family of non-trivial projective involutions acting on its projective tangent lines and fixing the tangency points. Suppose that its outer neighborhood admits a foliation by closed curves (including γ) such that the involution of each tangent line permutes its intersection points with every leaf. Then γ and the leaves are conics forming a pencil. We prove positive answer in the case, when the curve γ is C^4-smooth and the foliation admits a rational first integral. To this end, we show that each C^4-smooth germ γ of planar curve carrying a rationally integrable dual billiard structure is a conic and classify all the rationally integrable dual billiards on conics. They include the dual billiards induced by pencils of conics, two infinite series of exotic dual billiards and five more exotic ones.

10. Marie-Claude Arnaud

Title: Remainders of integrability for Hamiltonian dynamics on cotangent bundles

Abstract: For convex in the fiber Hamiltonian dynamics of cotangent bundles, there is a close relation between integrability, minimization properties and non-existence of conjugate vectors. Mather proved the existence of minimizing measures whose support has no conjugate points for the so-called Tonelli Hamiltonians, which can be seen as remainders of the invariant foliation of an integrable system. In a joint work with Anna Florio and Valentine Roos, we prove the existence of points with zero-asymptotic Maslov index without assuming the convexity of the Hamiltonian function.

11. Yael Karshon

Title: Geometric quantization of Lagrangian torus fibrations

Abstract: By the Arnold-Liouville's theorem, a regular completely integrable system
on a compact symplectic manifold M is Lagrangian torus fibration over an integral affine manifold B.
When M is pre-quantized, we give a simple proof that its Riemann-Roch number coincides with its number of Bohr-Sommerfeld leaves.
This can be viewed as an instance of the mysterious "independence of polarization" phenomenon of geometric quantization.
The proof uses the following fact, which on the surface has nothing to do with symplectic geometry, but which to our surprise we were able to prove only with the help of Lagrangian torus fibrations:
An integral affine structure on a manifold B determines a smooth measure on B; a so-called integral-integral affine structure determines a notion of integer points in B.
We show that, for a compact integral-integral affine manifold, the total volume is equal to the number of integer points.
This is joint work with Mark Hamilton and Takahiko Yoshida.

12. Alfonso Sorrentino

Title: On the persistence of periodic tori for symplectic twist maps

Abstract: In this talk I shall discuss the persistence of Lagrangian periodic tori for symplectic twist maps of the $2d$-dimensional annulus and the rigidity of completely integrable maps. This is based on a joint work with Marie-Claude Arnaud and Jessica Elisa Massetti.

13. Misha Shapiro

Title: Noncommutative integrability.
Abstract: We study a noncommutative generalization of pentagram map - a discrete dynamical system on the space of noncommutative weights of arcs of a directed graph embedded in a disk or cylinder. We show that the double quasi Poisson bracket between boundary measurements can be described via noncommutative r-matrix formalism. This gives a more conceptual proof of the result of N. Ovenhouse that traces of powers of Lax matrix form an infinite collection of noncommutative Hamiltonians in involution with respect to noncommutative Goldman bracket. This is a joint work with N.Ovenhouse and S.Artamonov.

14. Michael Gekhtman

Title: Twisted factorizations and multiple cluster structures in simple Lie groups
Abstract: We present a construction for cluster charts in simple Lie groups compatible with Poisson structures in the Belavin-Drinfeld classification. The key ingredient is a birational Poisson map from the group to itself that transform a Poisson bracket associated with a nontrivial Belavin-Drinfeld data into the standard one. It allows us to obtain a cluster chart as a pull-back of the Berenstein-Fomin-Zelevinsky cluster coordinates on the open double Bruhat cell. This is a joint work with M. Shapiro and A. Vainshtein.

15. Vadim Kaloshin

Title: Two and three caustics do not coexist for nearly circular domains.

Abstract: In this talk, I will discuss the question of co-preservation of two rational caustics by nearly circular billiard tables. Call a p-caustic a rational caustic of rotation number 1/p. It turns out 2-caustic and 3-caustic can't coexist for nearly circular non-circular domains. This is a partial answer to a conjecture by Tabachnikov. We shall also discuss the question of coexistence of p-caustic and q-caustic for p,q>2 will also be discussed.This is based on a joint work with Edmond Koudjinan.

16. Andrey E. Mironov.

Title. Finite gap differential operators and their discretizations.

Abstract: We shall discuss a connection between commuting ordinary differential operators and commuting difference operators.
In particular, we construct a discretization of the Lamé operator that preserves the spectral curve.

17. Alexander Mikhailov

Title: Quantisation of free associative dynamical systems.

Abstract: Traditional quantisation theories start with classical Hamiltonian systems with variables taking values in commutative algebras and then study their non-commutative deformations, such that the commutators of observables tend to the corresponding Poisson brackets as the (Planck) constant of deformation goes to zero. I propose to depart from dynamical systems defined on a free associative algebra A. In this approach the quantisation problem is reduced to the problem of finding of a two-sided ideal $J \subset A$ satisfying two conditions: the ideal J has to be invariant with respect to the dynamics of the system and to define a complete set of commutation relations in the quotient algebra [1].

To illustrate this approach I'll consider the quantisation problem for the Volterra family of integrable systems. In particular, I will show that odd degree symmetries of the Volterra chain admit two quantisations, one of them is a standard deformation quantisation of the Volterra chain, and another one is new and not a deformation quantisation. The periodic Volterra chain admits bi-Hamiltonian and bi-quantum structures [2]. The method of quantisation based on the concept of quantisation ideals proved to be successful for quantisation of stationary Korteveg-de–Vries hierarchies [3] which admits bi-quantum structure, the Toda hierarchy which admits non-deformation quantisation and other dynamical systems.

References

[1] A.V. Mikhailov. Quantisation ideals of nonabelian integrable systems. *Russian Mathematical Surveys*, 75(5):978–980, 2020.

[2] S. Carpentier, A.V. Mikhailov, J.P.Wang Quantisations of the Volterra hierarchy. *arXiv:2204.03095 [nlin.SI], 2022*

[3] V.M. Buchstaber, A.V. Mikhailov, KdV hierarchies and quantum Novikov's equations. *arXiv:2109.06357v2 [nlin.SI], 2021*

18. Andreas D Vollmer

Title: AndreThree-dimensional Levi-Civita metrics with projective vector fields.
Abstract :Projective vector fields are infinitesimal symmetries which preserve geodesics as unparametrised curves.
In 1882 Sophus Lie posed the problem of describing all surfaces admitting an algebra of projective vector fields of dimension one or higher. This problem was solved in recent years (by various authors).
This talk is concerned with the analogous 3-dimensional problem. Explicit normal forms are going to be presented for Riemannian metrics with projective vector fields, and, more generally, for Levi-Civita metrics of arbitrary signature.(Joint research with G. Manno, arXiv:2110.06785)

19. Bjorn Berntson

Title: The Soliton-Calogero-Moser correspondence with spin
Abstract: There is a well-established correspondence between soliton equations and many-body systems of Calogero-Moser type, whereby special solutions of the former are described by the latter. I will discuss recent work on applying this idea to systems with spin degrees of freedom and describe how this yields new insights on both sides of the correspondence. Specifically, I will present spin generalizations of the Benjamin-Ono equation and show that they have soliton solutions described by spin Calogero-Moser systems due to Gibbons and Hermsen. Of particular interest is this relation at the elliptic level, where the relevant soliton equation (the periodic spin non-chiral intermediate long wave equation) is related to the elliptic spin Calogero-Moser system via a novel Bäcklund transformation for the latter.
This talk is based on results from collaborations with Rob Klabbers, Edwin Langmann and Jonatan Lenells.

20. Maxim Arnold

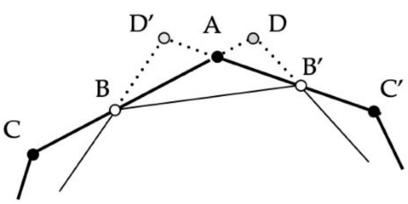
Title: Integrable transformations of centroaffine polygons.
Abstract: Let us call two centroaffine n-gons P and Q a-related if the

area of the parallelogram spanned by the radius-vectors of j-th vertices of P and Q does not depend on j. Generically, such a relation is 2-to-2. Thus, starting with the pair of a-related polygons P1 and Q1 one can construct unique polygon P2, a-related to Q1, but different from P1. Mapping P1 to P2 provides dynamics on the set of centroaffine
polygons. I will report on our findings in regards to complete integrability of such dynamics. The talk is based on joint work with D. Fuchs, S. Tabachnikov and A. Izosimov.

21. Gil Bor

Title: Dancing pairs, rolling balls and the Cartan distribution

Abstract: (Joint work with Luis Hernández Lamoneda.) A pair of polygons in the projective plane is “dancing” if one is inscribed in the other and they satisfy a certain cross-ratio relation at each vertex of the circumscribing polygon, as in the left figure. The right figure shows a dancing pair of hexagons.



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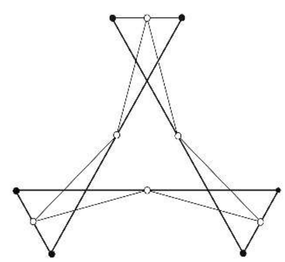

Dancing pairs correspond to trajectories of a non-holonomic mechanical system, consisting of rolling a ball, without slipping and twisting, along a polygon drawn on the surface of a sphere 3 times larger than the rolling ball. For example, the above dancing pair of hexagons corresponds to rolling twice around the perimeter of a right-angled equilateral triangle (an “octant”). The correspondence stems from reformulating both systems as piecewise rigid curves of a certain remarkable rank 2 non-integrable distribution defined on a 5-dimensional quadric in $\mathbf{RP}^6$, introduced by Élie Cartan in his 1894 thesis.

22. Krzysztof Marciniak

Title: Deformation of Stäckel systems into Painlevé-type systems and related non-autonomous soliton hierarchies

Abstract:
Stäckel systems (a broad class of integrable Hamiltonian systems that admit separation of variables and possess separation relations affine in the Hamiltonians) have a very rich geometric structure. In this talk I first explain this structure and then I show how to use this structure to obtain a systematic procedure of deforming of Stäckel systems, defined on a $2n$-dimensional Poisson manifold (M, π), into non-autonomous (explicitly multi-time dependent) Hamiltonian systems that are Frobenius integrable in the sense that their vector fields $Y_r(\xi, t) = \pi dH_r(\xi, t)$ satisfy the Frobenius integrability condition $\partial Yr/\partial ts - \partial Ys/\partial tr + [Yr, Ys] = 0$, meaning that there exists a common, multi-time solution of all the systems (at least locally defined) through each point ξ of the manifold M [1, 2]. Further, I will demonstrate that while the original Stäckel systems have a usual Lax representation, the obtained systems have an isomonodromic Lax representation which proves that they indeed are of Painlevé -type [3]. Finally, I will argue that it is possible to relate, in a very natural way, non-autonomous soliton hierarchies (like non-autonomous KdV or non-autonomous DWW) with the corresponding Painlevé -type systems, as it is possible to relate autonomous KdV or DWW with the appropriate Stäckel systems.

[1] Błaszak M., Marciniak K., Sergyeyev A., *Deforming Lie algebras to Frobenius integrable non- autonomous Hamiltonian systems*, Rep. Math, Phys. **87** (2021), No. 2, pp. 249-263

[2] Błaszak M., Marciniak K. and Domanski Z. *Systematic construction of non-autonomous Hamiltonian equations of Painlevé-type. I. Frobenius Integrability*, Stud. Appl. Mat. 2021; 1-43. DOI: 10.1111/sapm.12473

[3] Błaszak M., Domanski Z. and Marciniak K., *Systematic construction of non-autonomous Hamiltonian equations of Painlevé-type. II. Isomonodromic Lax representation*, to appear in Stud. Appl. Math. 2022: 1-52, DOI: 10.1111/sapm.12495

23. Holger Dullin

Title: A Tale of Two Polytopes related to geodesic flows on spheres
joint work with Diana Nguyen, Sean Dawson, University of Sydney

Abstract: Separation of variables for the geodesic flows on round spheres leads to a large family of integrable systems whose integrals are defined through the separation constants. Reduction by the periodic geodesic flow leads to integrable systems on Grassmanians. Specifically for the geodesic flown on the round S^3 the reduced system defines a family of
integrable systems on $S^2\times S^2$. We show that the image of these systems under a continuous momentum map defined through the action variables has a triangle as its image. The image is rigid and does not change when the integrable system is changed within the family. Each member of the family can be identified with a point inside a Stasheff polytope. Corners of the polytope correspond to toric systems (possibly with degenerations), edges correspond semi-toric systems (in various meanings of the word), and the face corresponds to ``generic'' integrable systems. A fundamental difference of this momentum map to that of a toric or semi-toric system is that the number of tori in the preimage of a non-critical point may be 1, 2, or 4.
The momentum map is continuous but not smooth along the images of hyperbolic singularities. The corresponding quantum problem and generalisations to higher dimensional spheres will be discussed.

24. Sonja Hohloch

Title: Extending compact Hamiltonian S^1-actions to integrable systems with mild degeneracies in dimension four.

Abstract:

Let (M, ω) be a 4-dimensional compact symplectic manifold and assume that the Hamiltonian J : M --› R induces an effective Hamiltonian S^1-action on (M, ω). Such a triple (M, ω, J) was called a *Hamiltonian S^1-space* by Karshon who classified these spaces by means of labeled directed graphs. Karshon also showed that some S^1-actions can be extended to a Hamiltonian S^1 x S^1-action on (M, ω) induced by a smooth integrable `toric' system (J, T) : (M, ω) --› R^2 and she gave the precise conditions under which this is possible.

Thus, if one wants to extend a larger class of Hamiltonian S^1-spaces to an integrable system one needs to admit a larger class than toric systems as target. Hohloch & Sabatini & Sepe & Symington's work in progress removes part of Karshon's conditions and then they can extend to so-called *semitoric integrable systems* (J, S) : (M, ω) --› R^2 which are, roughly, integrable systems that only allow elliptic-regular, elliptic-elliptic, and/or focus-focus points as singularities. These systems induce in fact an S^1 x R^1-action on M, thus the name.

In this talk, we define the class of so-called *hypersemitoric integrable systems*, which is the `easiest, nicest, and smallest' set of smooth integrable systems (J, H) : (M, ω) --› R^2 to which *all* Hamiltonian S^1-spaces (M, ω, J) can be extended. These systems induce an S^1 x R^1-action on M and the *nondegenerate* singularities of (J, H) may have components of regular, elliptic, focus-focus, and/or hyperbolic type, thus the name. The occurence of (finitely many) degenerate singular points cannot always be avoided, but there is a certain choice of what kind of degeneracies one wants to admit in the extension. Since parabolic (`cuspidal') points are a very natural and `most nondegenerate' class of degenerate points we opted to admit this type of degeneracies. Eventually, we will outline the construction of extending a Hamiltonian S^1-space to a hypersemitoric system by means of an example.

This talk is based on a joint work with Joseph Palmer, see arXiv:2105.00523.

25. Igor Zelenko

Title: Projective and affine equivalence of sub-Riemannian metrics: integrability, generic rigidity, the Weyl type theorems, and separation of variables conjecture.

Abstract: Sub-Riemannian metrics are defined by a distribution (a subbundle of the tangent bundle) together with a Euclidean structure on each fiber. The Riemannian metrics correspond to the case when the distribution is the whole tangent bundle. Two sub-Riemannian metrics are called projectively equivalent if they have the same geodesics up to a reparameterization and affinely equivalent if they have the same geodesics up to affine reparameterization. In the Riemannian case, both equivalence problems are classical: local classifications of projectively and affinely equivalent Riemannian metrics were established by Levi-Civita in 1898 and Eisenhart in 1923, respectively. In particular, a Riemannian metric admitting a nontrivial (i.e. non-constant proportional) affinely equivalent metric must be a product of two Riemannian metrics i.e. separation of variables (the de Rham decomposition) occur, while for

the analogous property in the projectively equivalent case a more involved (``twisted") product structure is necessary. The latter is also related to the existence of sufficiently many commuting nontrivial integrals quadratic with respect to velocities for the corresponding geodesic flow. We will describe the recent progress toward the generalization of these classical results to sub-Riemannian metrics. In particular, we will discuss the genericity of metrics that do not admit non-constantly proportional affinely/projectively equivalent metrics and the separation of variables on the level of linearization of geodesic flows (i.e. on the level of the Jacobi equations) for metrics that admit non-constantly proportional affinely equivalent metrics. We also describe the sub-Riemannian analog of the Weyl theorem that all metrics that are simultaneously projectively equivalent and conformal are constantly proportional. The talk is based on the collaboration with Frederic Jean (ENSTA, Paris), Sofya Maslovskaya (INRIA, Sophia Antipolis), and Zaifeng Lin (Texas A&M University)