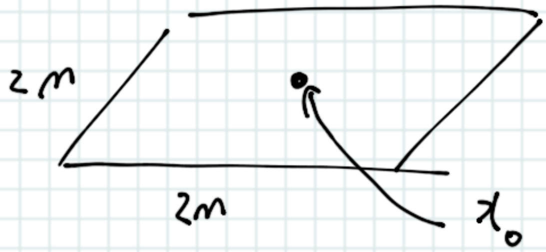
Theorem 1 (with Avelio)

if $T < T_{rec}^-$, then one CAN recover ϕ given $\phi\left(\frac{2\pi}{T}\right)$

① 

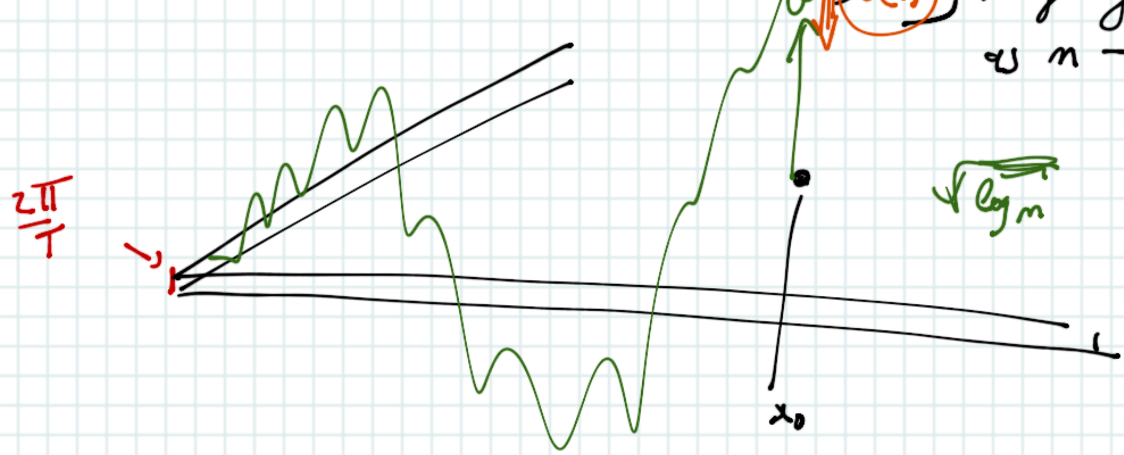
ϕ_n 0-Bdy GFF on $\Lambda_n = \{-n,n\}^2$

x_0 in the "Bulk"

$\exists$ recovery funct° $F_{x_0} = F_{x_0}(e^{iT\phi_n}) = F(\phi(\frac{2\pi}{T}))$

$$\mathbb{E}\left[\left(\phi_n(x_0) - F_{x_0}(e^{iT\phi_n})\right)^2\right] \leq C \lesssim e^{-c\frac{1}{T}}$$

$O(1)$ uniformly as $n \to +\infty$



② if $T < T_{rec}^-$ $\quad f : \Lambda_n \to \mathbb{R}$ "smooth"

$$\mathbb{E}\left[\left|\langle\phi_n, f\rangle - F_f(e^{iT\phi_n})\right|^2\right] = o(1)_{n\to+\infty}$$

Theorem 2 (w Avelio)

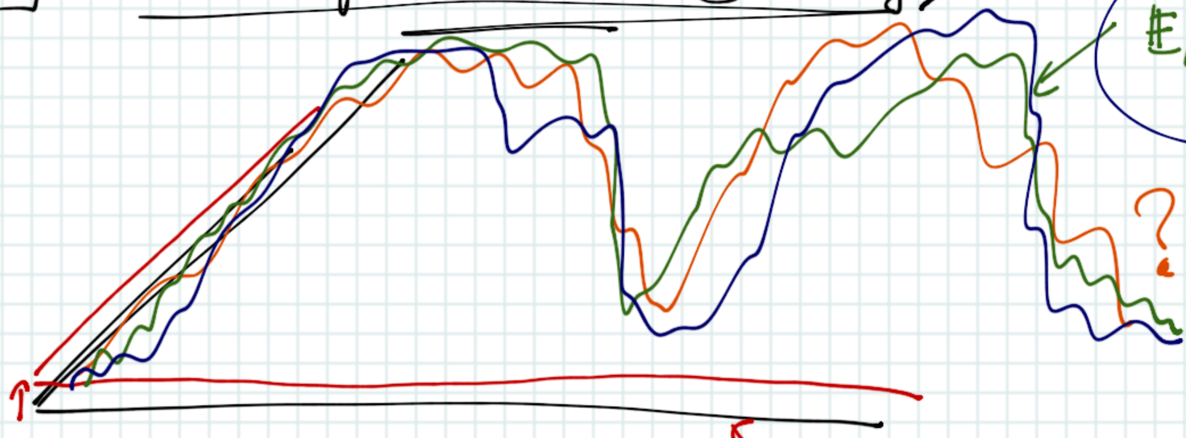
$T > T_{rec}^+$ $\left(T_{rec}^- = T_{rec}^+ = \text{Spectral Height Gap GFF} \right)$

∀ reconstruction funct F_{x_0} $\quad C = C_T > 0$

$$\mathbb{E}\left[\left(\phi_n(x_0) - F_{x_0}\left(e^{iT\psi_n}\right)\right)^2\right] \geq C \log n$$

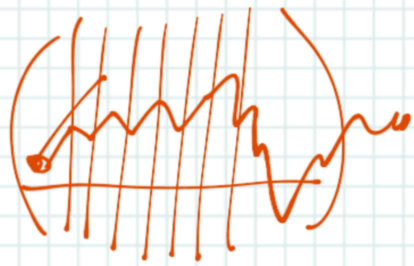
uniformly in n

Sketch/ low-temperature phase (Recovery)



① $a = \phi_n\left(\frac{2\pi}{T}\right)$

② ∃! ground state $\Psi_{n,0}$, Argmin $\phi \in a - \frac{2\pi}{T}\mathbb{Z}$



if $a \equiv 0$

↓

$\Psi_{n,0}$ = flat

a Q. Disorder

~~Quenched Peierls Argument~~ ??

Pirogov - Sinai

③ The conditional Expectation $\left[x \mapsto \mathbb{E}\left[\phi_n(x) \,\middle|\, e^{iT\psi_n}\right]\right]$

(4) The initial field ϕ_m

Annealed Peierls Argument

$$\boxed{F_{x_0}\left(e^{iT\phi_m}\right) = \mathbb{E}\left[\phi_m(x_0) \,\middle|\, e^{iT\phi_m}\right]}$$

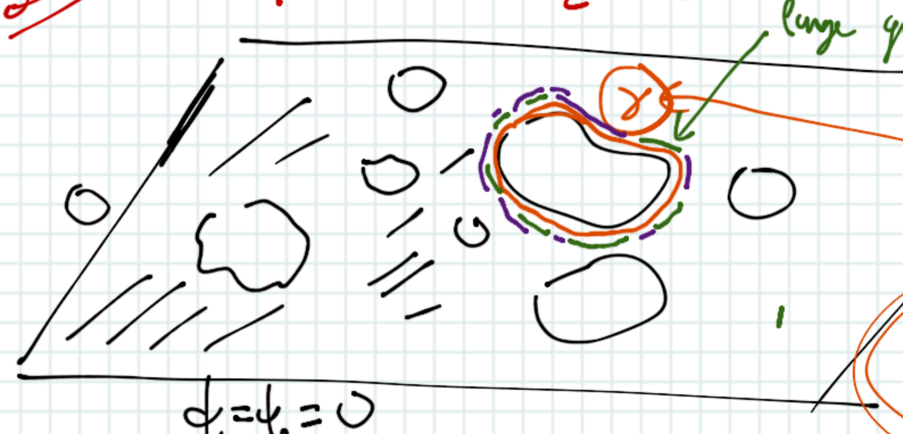
$$\mathbb{E}\left[\left(\phi_m(x_0) - \mathbb{E}\left[\phi_m(x_0)\,\middle|\, e^{iT\phi_m}\right]\right)^2\right]$$

$$= \frac{1}{2}\,\mathbb{E}\left[\left(\phi_1(x_0) - \phi_2(x_0)\right)^2\right]$$

ϕ_1, ϕ_2 are $\sim \mathcal{L}(\phi \mid e^{iT\phi})$

conditionally ind given $e^{iT\phi}$

goal $\quad \phi_1(x_0) \approx \phi_2(x_0)$ with High Probability



Annealed Peierls Argument

Along γ

$$\left|\nabla(\phi_1 - \phi_2)\right| \gtrsim \frac{2\pi}{T}$$

$$\Downarrow$$

$$\frac{\pi}{T}$$

$\phi_1 \sim GFF$

$\phi_2 \sim GFF$

$\boxed{\text{Large Gradients are Costly for the GFF}}$

$$\boxed{\mathbb{P}\left[\,|\nabla\phi_e| \geq u \quad \forall e \in F\,\right] \lesssim \exp\left(-C\,u^2\,|F|\right)}$$

High temperature Case

Generalise Fröhlich-Spencer Proof of Delocalisation of iV-GFF to this setting with Quenched-Disorder

Lack of Symmetry

Riemann-Theta-functions