A MARSTRAND THEOREM FOR SUBSETS OF INTEGERS

1. Abstract

A classical result of Marstrand states: if K_1, K_2 are Borel subsets of $\mathbb{R}$, then

$$\dim_H(K_1 + \lambda K_2) = \min\{1, \dim_H(K_1) + \dim_H(K_2)\}$$

for Lebesgue almost every $\lambda \in \mathbb{R}$ (here $\dim_H(X) =$ Hausdorff dimension of X).

In this talk we suggest a counting dimension $\dim_{\mathbb{Z}}$ for subsets of integers and prove that, under suitable conditions on two subsets $E, F \subset \mathbb{Z}$,

$$\dim_{\mathbb{Z}}(E + \lfloor \lambda F \rfloor) = \min\{1, \dim_{\mathbb{Z}}(E) + \dim_{\mathbb{Z}}(F)\}$$

for Lebesgue almost every $\lambda \in \mathbb{R}$. The result has direct consequences when applied to arithmetic sets, such as the integer values of a polynomial with integer coefficients. Joint work with Carlos Gustavo Moreira.