

SIEVE THEORY 2015
ASSIGNMENT 4
DUE DATE: WEDNESDAY, JUNE 17, 2015

Exercise 1. Let $q > 1$, and $1 \leq a < q$ be coprime to q . Show that

$$\sum_{\substack{p \leq x \\ p = a \pmod{q}}} \frac{\log p}{p} = \frac{1}{\varphi(q)} \log x + O(1)$$

(the sum over primes). You may assume the Prime Number Theorem in arithmetic progressions in the form

$$\psi(x; q, a) := \sum_{\substack{n \leq x \\ n = a \pmod{q}}} \Lambda(n) = \frac{x}{\varphi(q)} + O_q(x \exp(-\sqrt{\log x})).$$

Exercise 2. Show that the number of primes $p \leq x$ so that $2p + 1$ is also prime is at most $O(x/(\log x)^2)$.