Graph Theory

Homework assignment #2

Due date: Sunday, December 6, 2015

Problem 1. Prove that every two paths of maximum length in a connected graph must have a vertex in common.

Problem 2. Let Q_k be the k-dimensional hypercube graph defined as follows:

- $V(Q_k) = \{0,1\}^k$,
- $E(Q_k) = \big\{\{(x_i)_{i=1}^k, (y_i)_{i=1}^k\} \colon (x_i) \text{ and } (y_i) \text{ differ in exactly one coordinate}\big\}$.

Prove that $\kappa(G) = \kappa'(G) = k$.

Problem 3. Prove that a graph is 2-connected if and only if for any three vertices x, y, and z, there is a path from x to z that passes through y.

Problem 4. Let G be a 3-regular graph. Prove that $\kappa(G) = \kappa'(G)$.

Problem 5. Show that every k-connected graph with at least $2k$ vertices contains a cycle of length at least $2k$.

Problem 6. Suppose that every pair of vertices of a graph G has an odd number of common neighbors. Prove that G is Eulerian.

Problem 7. Let G be a connected graph with n vertices. Prove that G contains a path of length $\min\{2\delta(G), n-1\}$.

Problem 8. Prove that the maximum number of edges in a non-Hamiltonian graph with n vertices is $\binom{n-1}{2} + 1$.

Please do NOT submit written solutions to the following exercises:

Exercise 1. Prove that G contains the path of length two as an induced subgraph if and only if G is not a union of vertex-disjoint complete graphs.