# Assignment 3 - Geometric Optimization (0368-4144)

Due: December 24, 2013

## Problem 1

Explain the behavior of the subexponential LP algorithm on the following set $H$ of $2d$ constraints in $\mathbb{R}^d$ (which define a hypercube):

$$1 \le x_i \le 2, \qquad i = 1, \ldots, d,$$

where we wish to minimize the objective function $\frac{1}{3}x_1 + \frac{1}{3^2}x_2 + \cdots + \frac{1}{3^d}x_d$. (In addition, we have the set $H^+$ of limiting constraints $x_i \ge 0$, for $i = 1, \ldots, d$.)

Address issues like: What is the form of a basis $B$? How does the basis recomputation routine $\texttt{basis}(B, h)$ work? What is the hidden dimension of a basis $B$ within a set $G \subseteq H$ of constraints? Explain explicitly why the hidden dimension always becomes smaller during the execution of the algorithm. Describe the derivation of a recurrence for the expected running time in this special case.

## Problem 2

Explain in detail why the following problems are LP-type: Define the set $H$ of constraints, the set $\mathcal{W}$ of values, the function $w$ from subsets of constraints to $\mathcal{W}$, explain why $w$ is monotone and local, and show an upper bound on the maximum size of a basis (the combinatorial dimension of the problem). Discuss whether the problem is basis-regular or not.

(a) Given two disjoint convex polytopes $P, Q$ in $\mathbb{R}^d$, each specified as the convex hull of $n$ points, find $p \in P$, $q \in Q$ that minimize the distance $d(p, q)$. ($p$ and $q$ need not be vertices of the polytopes!)

(b) Given a set $P$ of $n$ points on a line, find four intervals of the same length $w$ whose union contains $P$, for which $w$ is as small as possible. (Show, as a preliminary step, that $P$ can be covered by four intervals of length $w$ if and only if every 5 points of $P$ can be covered by four such intervals (not necessarily the same).)

## Problem 3

Show that a set of axis-parallel rectangles is pierced by 3 points if and only if every 16 of them are pierced by three points.

## Problem 4

For a fixed integer $p$, assume that we have a decision procedure $D_p$ that tests whether $n$ axis-parallel rectangles in the plane can be pierced by $p$ points, which runs in at most $D_p(n)$ time.

**(a)** Show that the rectilinear $p$-center problem can be solved in time $O(D_p(n) \log n)$ using monotone matrix searching.

**(b)** Show that the rectilinear $p$-center problem can be solved in randomized expected time $O(D_p(n))$ using Chan's technique. Conclude that the cases $p = 2, 3$ (resp. $p = 4$) have $O(n)$ (resp., $O(n \log n)$) expected running time. (**Note:** It's OK to consult Chan's paper, but no copying: Explain and fill in the details in your own way.)