Appendix: If spaces are not a joy to you

If spaces are not a joy to you, feel free to switch to the language of invariance under transformations, as follows.

For single-space notions, such as "bounded set", "continuous function", "simple path".

well-defined in *Euclidean vector* ^{fd}space	invariant under *orthogonal* transformations
well-defined in *Euclidean affine* ^{fd}space	invariant under *orthogonal affine* transformations
well-defined in *vector* ^{fd}space	invariant under *linear* transformations
well-defined in *affine* ^{fd}space	invariant under *affine* transformations

All transformations are assumed to be invertible.
Orthogonal transformations are linear isometries. $\forall x \ \ |Ax| = |x|$.
Orthogonal affine transformations are composed from orthogonal transformations and shifts. $T(x) = Ax + b, \quad \forall x \ \ |Ax| = |x|$.
Affine transformations are composed from linear transformations and shifts. $T(x) = Ax + b$.

Points: $x \mapsto T(x)$.
Sets: $X \mapsto T(X) = \{T(x) : x \in X\}; \qquad x \in X \iff T(x) \in T(X)$.
Paths: $\gamma \mapsto T \circ \gamma; \qquad \gamma(t) = x \iff (T \circ \gamma)(t) = T(x)$.
Functions: $f \mapsto f \circ T^{-1}; \qquad f(x) = (f \circ T^{-1})(T(x))$.

For mappings (linear and nonlinear) from a space to a space, and related two-space notions

well-defined for a pair of *Euclidean vector* ^{fd}spaces	invariant under pairs of *orthogonal* transformations
well-defined for a pair of *Euclidean affine* ^{fd}spaces	invariant under pairs of *orthogonal affine* transformations
well-defined for a pair of *vector* ^{fd}spaces	invariant under pairs of *linear* transformations
well-defined for a pair of *affine* ^{fd}spaces	invariant under pairs of *affine* transformations

Mappings: $f \mapsto T_2 \circ f \circ T_1^{-1}$;

$$y = f(x) \iff T_2(y) = (T_2 \circ f \circ T_1^{-1})(T_1(x)).$$