

## After the exam of 05.08.2016: Typical errors, comments etc.

At most 40 points (rather than 35) are given per question.

### QUESTION 1

ERROR: only one Lagrange multiplier:  $40 - 20 = 20$  points.

CLARIFICATION: in order to have only one Lagrange multiplier you need extremum on  $(n - 1)$ -dimensional manifold.

ERROR: wrong number of constraints:  $40 - 30 = 10$  points.

CLARIFICATION: the number of constraints  $n - k$  must correspond to the dimension  $k$  of the manifold.

ERROR: constraints appear without explanation:  $40 - 35 = 5$  points.

CLARIFICATION: you have to explain, why a manifold can be described by constraints.

FATAL ERRORS:<sup>1</sup>

Lagrange multipliers are not used (and the goal is not reached);  
wrong dimensions ( $\mathbb{R}^n$  instead of  $\mathbb{R}^{n-k}$ , or  $\mathbb{R}$  instead of  $\mathbb{R}^n$ , etc);  
 $g(x_0) = 0$ , but  $g(x) \neq 0$  for  $x \in M$  near  $x_0$ .

### QUESTION 2 FOR CONTINUOUS $f$

ERROR: bounded support of  $f$  is not used when proving the uniform convergence of  $f_\theta$  to  $f$  (as  $\theta \rightarrow 0$ ): -5 points.

CLARIFICATION: the convergence of  $(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$  to  $(x, y)$  is uniform on a bounded set (of points  $(x, y)$ ), not on the whole  $\mathbb{R}^2$ .

ERROR: uniform continuity of  $f$  is not used when proving the uniform convergence of  $f_\theta$  to  $f$  (as  $\theta \rightarrow 0$ ): -5 points.

ERROR: bounded support of  $f$  is not used when estimating  $\int |f_\theta - f|$ : -5 points.

ERROR:  $\max |f_\theta - f| \leq \max f_\theta - \max f$ : -15 points.

ERROR: no proof of the *uniform* convergence of  $f_\theta$  to  $f$ : -15 points.

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<sup>1</sup>It means, no points for this question!

QUESTION 2 FOR INTEGRABLE  $f$ 

ERROR:  $|f_\theta - f| \leq |h_\theta - g|$ : -2 points.

CLARIFICATION: we have  $g \leq f$  and  $f_\theta \leq h_\theta$ , therefore  $f_\theta - f \leq h_\theta - g \leq |h_\theta - g|$ ; however,  $f_\theta - f$  may be negative, in which case  $|f_\theta - f| = f - f_\theta$  may exceed  $|h_\theta - g|$ .

ERROR:  $|f_\theta - f| \leq |h_\theta - f| \leq |h_\theta - h|$ : -5 points.

ERROR: rotation invariance of integral used but not mentioned: -5 points.

FATAL ERRORS: a “proof” of evidently wrong claim, such as  $\int |f_\theta - f| = 0$  or  $f_\theta = f$ .

## QUESTION 3

ERROR: a 3-dimensional change of variables: -15 points.

CLARIFICATION: the function  $\theta$  need not be differentiable, thus  $\varphi$  need not be a diffeomorphism.

ERROR: (in addition to a 3-dimensional change of variables) the determinant is taken of a matrix of the form  $\begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}$ : -5 points.

CLARIFICATION:  $D\varphi$  is rather of the form  $\begin{pmatrix} * & * & * \\ * & * & * \\ 0 & 0 & 1 \end{pmatrix}$  (assuming differentiability of  $\theta$ , otherwise  $D\varphi$  does not exist).

## GRADES STATISTICS

Total	Question 1	Question 2	Question 3	Question 4
120	40	40		40
117	40	39		38
105		25	40	40
100	20	40	40	
90	20	30	40	
88	20	35	33	
75	40	35		0
70	20	30	20	
70	0	30	40	
70	20	25	25	
68		26	35	7
65	0	25	40	
65		20	40	5
65		25	40	0
65	40	20	5	
60	0	35	25	
60	5	30	25	
60	20	20	20	
60		25	35	0
60	20	15	25	

Total	Question 1	Question 2	Question 3	Question 4
48		28	20	0
45	0	15	30	
45	5	15	25	
45		15	30	0
45	0	15	30	
45	20	0	25	
45	20	5	20	
40	20	20	0	
40	0		40	0
40		10	0	30
30	10	0	20	
30	30	0		0
25		5	20	0
25	0	25		0
25		5	20	0
20		0	20	0
15		0	0	15
10		0	0	10
5	5		0	0
0		0	0	0