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On Optimal Allocation of Trunks in a Local Network of Public Telephone Exchanges

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ABSTRACT

The optimal allocation of trunks in a local network of public telephone exchanges of the step-by-step type is determined. The network is comprised of a set of "local" exchanges receiving calls from a set of "source" exchanges. The switching configuration is such that calls originating at a given source exchange are routed *randomly* to one of the various local exchanges rather than being transferred directly to their destinations. It is shown that, contrary to the common practice of partitioning the trunks leaving each source exchange *proportionately* to the distribution of offered load to the various local exchanges, the optimal allocation, which minimizes total flow in the system, is that one which for each source exchange directs *all* outgoing trunks to a *single* local exchange. This latter exchange is the one to which the offered load from the given source exchange is *maximal*, but distinct source exchanges may have different "maximal" local exchanges. This qualitative result may be shown to hold true for any monotone increasing concave objective function. The method of analysis may also prove useful for other studies of communications networks.

INTRODUCTION

We study a telephone network common to various sections of the Israeli telecommunication network. This network is comprised of a set of n local exchanges and a set of m source exchanges offering traffic to the local ones. The local exchanges are characterized by having the *same* first digit but a distinct second one. A call originating at a given source exchange and having as its destination local exchange j (i.e., an exchange whose second digit is j) is routed *randomly* to one of the n local exchanges. Typically, the call arrives first at local exchange i ($i \neq j$) and is then *retransferred* to its destination.

Each source exchange is connected to the various local exchanges by trunks which carry the load offered by this exchange. The total number of trunks allocated to carry this load is determined by *standard teletraffic methods* (*Siemens*, 1970) so as to *assure* a *desired grade-of-service* (defined as the probability of a busy line). The problem then is to find the *optimal* allocation of the trunks leaving a particular source exchange, among the various local exchanges.

It will be shown that, *contrary* to the common practice of allocating trunks *proportionately* to the distribution of offered load, the optimal allocation, which minimizes total flow in the system, is to direct *all* trunks from each source exchange to a *single* local exchange. This latter exchange is found to be the one which is being offered the largest load from the given source exchange. Distinct source exchanges may have different "optimal" local exchanges. This is a *qualitative* result which is independent of the total number of trunks leaving each source exchange. This qualitative result may be shown to hold for any monotone increasing concave function of the number of trunks connecting a source exchange to a local one. Obviously, the specific local exchange, to which all trunks from a given source exchange should be directed, may change according to the particular measure of effectiveness.

THE MODEL

Assume that there are m source exchanges offering traffic to n local exchanges. Let A_{ki} ($k = 1, 2, \ldots, m$; $i = 1, 2, \ldots, n$) be the offered traffic from source exchange S_k to local exchange E_i. A_{ki} is given in Erlangs and is assumed to be constant over the period under consideration. The total amount of traffic offered by S_k to all local exchanges is $A_{k\cdot} = \Sigma_{i=1}^{n} A_{ki}$.

Let N_{ki} be the number of trunks leading directly from S_k to E_i. The total number of trunks carrying traffic from S_k to all n local exchanges is $N_{k\cdot} = \Sigma_{i=1}^{n} N_{ki}$. Usually, in such a network, the value of $N_{k\cdot}$ is given, i.e., $N_{k\cdot}$ is determined from teletraffic tables (*Siemens*, 1970) as a function of the total offered load $A_{k\cdot}$ and the grade-of-service standards specified by the management.

Let Y_{ki} be the traffic carried from S_k to E_i. The total traffic carried through all N_k trunks leading from S_k to the local exchanges is $Y_{k\cdot} = \Sigma_{i=1}^{n} Y_{ki}$.

In addition to the traffic offered by the source exchanges to the local exchanges, each local exchange generates calls to its sister exchanges. This internal traffic, which is routed directly between the exchanges, is independent of the A_{ki}'s. Consequently this flow does not affect the determination of the N_{ki}'s and will not be considered in the sequel.

A schematic network with m source exchanges and three local exchanges is drawn in Fig. 1 (the W_{ij}'s are defined subsequently).

Let $q_{ki} = N_{ki}/N_{k\cdot}$ ($k = 1, 2, \ldots, m$; $i = 1, 2, \ldots, n$). Assuming that the calls generated at S_k are distributed proportionately among all N_k trunks leaving S_k, then q_{ki} is the probability that a call offered from S_k will arrive at E_i. This probability is independent of the destination of the call, which may be either E_i or E_j, $j \neq i$. If the destination is E_j then the call is rerouted from E_i to E_j.

The proportional distribution of calls among the N_k trunks implies that

$$Y_{ki} = A_{k\cdot} q_{ki}. \quad (1)$$

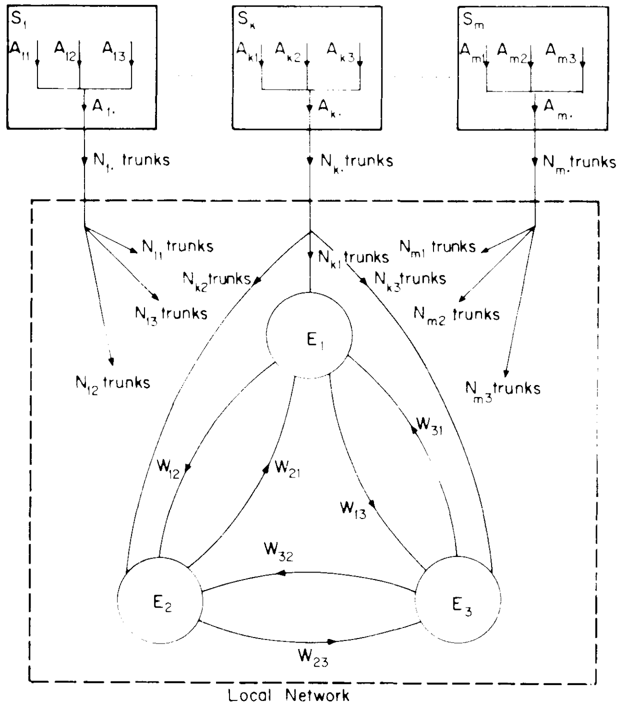

Fig. 1. A scheme of the network.

Hence, the total traffic from the source-exchange arriving at E_i is

$$Y_{\cdot i} = \sum_{k=1}^{m} Y_{ki} = \sum_{k=1}^{m} A_{k\cdot} q_{ki}.$$

Now, part of the traffic arriving at E retransferred to its final destination. Let W_{ij} ($i, j =$ 1, 2, ..., n; $i \neq j$) be the amount of traffic from E_i to E_j. Then

$$W_{ij} = \sum_{k=1}^{m} A_{kj} q_{ki}.$$

The reasoning behind (3) is that from the A_{kj} Erlangs offered by S_k to E_j, a prop calls arrives first at E_i and has to be re- Summing over all m source-exchange Eq. (3). Clearly, the total traffic arriv all source-exchanges is, using (2),

$$\sum_{j=1}^{n} \sum_{k=1}^{m} A_{kj} q_{ki} = \sum_{k=1}^{m} A_{k\cdot} q_{ki} =$$

and the total amount of traffic flow branches of the entire network is exchange, to which all trunks from a given source exchange should be directed, may change according to the particular measure of effectiveness.

THE MODEL

Assume that there are m source exchanges offering traffic to n local exchanges. Let A_{ki} ($k = 1,2,\ldots,m$; $i = 1,2,\ldots,n$) be the offered traffic from source exchange S_k to local exchange E_i. A_{ki} is given in Erlangs and is assumed to be constant over the period under consideration. The total amount of traffic offered by S_k to all local exchanges is $A_{k\cdot} = \sum_{i=1}^{n} A_{ki}$.

Let N_{ki} be the number of trunks leading directly from S_k to E_i. The total number of trunks carrying traffic from S_k to all n local exchanges is $N_{k\cdot} = \sum_{i=1}^{n} N_{ki}$. Usually, in such a network, the value of $N_{k\cdot}$ is given, i.e., $N_{k\cdot}$ is determined from teletraffic tables (*Siemens*, 1970) as a function of the total offered load $A_{k\cdot}$ and the grade-of-service standards specified by the management.

Let Y_{ki} be the traffic carried from S_k to E_i. The total traffic carried through all N_k trunks leading from S_k to the local exchanges is $Y_{k\cdot} = \sum_{i=1}^{n} Y_{ki}$.

In addition to the traffic offered by the source exchanges to the local exchanges, each local exchange generates calls to its sister exchanges. This internal traffic, which is routed directly between the exchanges, is independent of the A_{ki}'s. Consequently this flow does not affect the determination of the N_{ki}'s and will not be considered in the sequel.

A schematic network with m source exchanges and three local exchanges is drawn in Fig. 1 (the W_{ij}'s are defined subsequently).

Let $q_{ki} = N_{ki}/N_k$ ($k = 1,2,\ldots,m$; $i = 1,2,\ldots,n$). Assuming that the calls generated at S_k are distributed proportionately among all N_k trunks leaving S_k, then q_{ki} is the probability that a call offered from S_k will arrive at E_i. This probability is independent of the destination of the call, which may be either E_i or E_j, $j \neq i$. If the destination is E_j then the call is rerouted from E_i to E_j.

The proportional distribution of calls among the $N_{k\cdot}$ trunks implies that

$$Y_{ki} = A_{k\cdot} q_{ki}. \qquad (1)$$

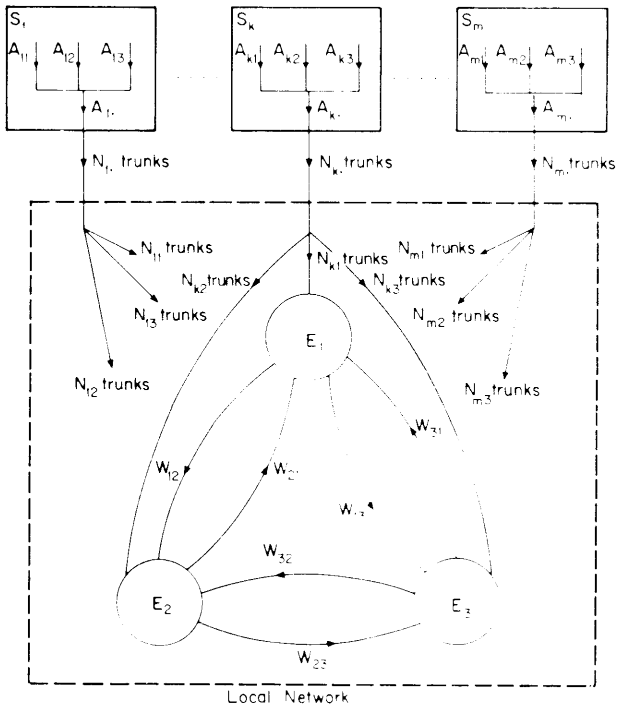

Fig. 1. A scheme of the network.

Hence, the total traffic from the source-exchanges arriving at E_i is

$$Y_{\cdot i} = \sum_{k=1}^{m} Y_{ki} = \sum_{k=1}^{m} A_{k\cdot} q_{ki}. \qquad (2)$$

Now, part of the traffic arriving at E_i has to be retransferred to its final destination. Let W_{ij} ($i,j = 1,2,\ldots, n$; $i \neq j$) be the amount of traffic rerouted from E_i to E_j. Then

$$W_{ij} = \sum_{k=1}^{m} A_{kj} q_{ki}. \qquad (3)$$

The reasoning behind (3) is that from the traffic of A_{kj} Erlangs offered by S_k to E_j, a proportion of q_{ki} calls arrives first at E_i and has to be rerouted to E_j. Summing over all m source-exchanges we obtain Eq. (3). Clearly, the total traffic arriving at E_i from *all* source-exchanges is, using (2),

$$\sum_{j=1}^{n} \sum_{k=1}^{m} A_{kj} q_{ki} = \sum_{k=1}^{m} A_{k\cdot} q_{ki} = Y_{\cdot i}, \qquad (4)$$

and the total amount of traffic flowing through all branches of the entire network is given by changes which are offered approximately the same load, little harm will be caused if we allocate the trunks proportionately to the distribution of the offered load. Such a design will also be insensitive to any small changes in that distribution. However, if a single local exchange is heavily loaded relative to its sister exchanges, all trunks should be directed to it. Obviously, in the case of a drastic and unpredicted future change in the level of traffic between exchanges, a new allocation would have to be made.

Finally, as an example, let us find the relative "saving" in traffic when using the optimal allocation as opposed to the proportional allocation for the case of a single source exchange and two local ones. Let $A_{k,i(k)} = pA_{k\cdot}$ $(0.5 \leqq p \leqq 1)$. Then $R/A_{k\cdot} = (2p-1)(1-p)$. Clearly, $R/A_{k\cdot} = 0$ for $p = 0.5$ or 1, and for $p = 0.75$, $R/A_{k\cdot} = 0.125$ is *maximal*.

CONCLUSION

We wish to emphasize two points:

(i) The above results have applications in both existing and planned networks. In an existing network, an optimal re-allocation of trunks, subject to practical engineering constraints, will reduce the flow in the system and thus result in a better grade-of-service. In planned networks, applying optimal allocation will minimize the total investment needed for the system.

(ii) The technique of analysis used here was to transform a typical queueing problem into a linear programming problem which then yielded a simple and easily employed solution. We feel that this technique may be profitably extended to other studies of telecommunication networks.

ACKNOWLEDGEMENT

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