Elevator-Type Polling Systems *

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November 1992

Abstract

The basic polling system is a configuration of N queues attended by a single server, usually in a *cyclic* order. In an Elevator-type polling scheme, instead of moving in a cyclic fashion, the server scans the channels back and forth, residing in each queue for a duration of time determined by the service discipline. Such systems have a wide variety of applications in the areas of telecommunications, computer networks, manufacturing, maintenance and repair, etc.

In this work we apply a unified approach to study four Elevator-type polling systems, distinguished by their service regimes. The system-models are called Elevator Exhaustive, Elevator Gated, Elevator Globally-Quasi-Exhaustive and Elevator Globally-Gated (the Exhaustive, Gated and Globally-Gated disciplines were partially studied in the literature). For each system we provide a comprehensive analyses regarding cycle times in the up and down directions, server's sojourn times in the various channels in each direction, customers' waiting times, and channels' queue sizes. Furthermore, we derive conditions under which the durations of the up and down cycles are the same. The calculation of customers' mean waiting times are based on a derivation of a general relation-formula that can be used in conjunction with many other polling schemes.

*Supported by a Grant from the France-Israel Scientific Cooperation (in Computer Science and Engineering) between the French Ministry of Research and Technology and the Israeli Ministry of Science and Technology, Grant Number 3321190.

1 Introduction

Queueing systems consisting of N queues (channels) served by a single server which incurs switch-over periods when moving from one queue to another have been widely studied in the literature and used as a central model for the analysis of a wide variety of applications in the areas of telecommunications, computer networks, manufacturing, etc. Very often such applications are modeled as a polling system in which the server visits the queues in a cyclic order or according to some polling table (c.f. Baker & Rubin [1987]).

In many of these applications, as well as in most polling models, it is common to control the amount of service given to each queue during the server's visit. Two widely used policies are the Exhaustive and the Gated regimes, whose analysis (for queues with infinite buffers) has been extensively studied in the literature (see Takagi [1986] and [1990]). Recently, the Globally-Gated and the Globally-Quasi-Exhaustive service regimes were proposed by Boxma, Levy & Yechiali [1992], who provided a thorough analysis of the cyclic Globally-Gated scheme. Boxma, Westsrate and Yechiali [1993] further extended the Globally-Gated model to include server interruptions, and applied it to a real-world repairman problem. A Globally-type regime uses a time-stamp mechanism for its cyclic reservation: the server performs a Hamiltonian tour through the queues, and uses the instant of cycle-beginning as a reference point of time. If $(n_1, n_2, \ldots, n_N)$ is the state-vector of the number of customers present at the various queues at the start of the Hamiltonian tour (when all gates are globally closed) then, under the Globally-Gated regime, the server serves exactly n_i customers at queue i, whereas under the Globally-Quasi-Exhaustive discipline, the server resides in queue i for the duration of n_i ordinary busy periods.

In this paper we concentrate on the Elevator-Type (scan) polling scheme: instead of moving cyclically through the channels, the server first serves the channels in one direction, i.e. in the order $1, 2, ..., N$ ('up' cycle) and then reverses its orientation and serves the channels in the opposite direction ('down' cycle), i.e. going through channels $N, N-1, ..., 1$. Then it changes direction again, and keeps moving in this manner back and forth. This type of polling mechanism is encountered in many applications, e.g. it models a common scheme of addressing a hard disk for writing (or reading) information on (or from) different tracks. The Elevator scheme 'saves' the return walking time from channel N to channel 1 (when compared to cyclic polling systems).

Additional motivation for considering Elevator-type models emerges from the results obtained by Browne & Yechiali [1989] when studying "Dynamic Priority Rules for Cyclic-Type Queues". In that paper it is shown that the total time to complete a Hamiltonian tour of the N channels, when the visit order is given by the permutation $\pi = (\pi_{(1)}, \pi_{(2)}, ..., \pi_{(N)})$, is

$$E(C) = \sum_{i=1}^{N} a_{\pi(i)} \prod_{r=i+1}^{N} (1 + \alpha_{\pi(r)}), \tag{1}$$

where a_i is the initial 'core' of work at channel i, and α_i is the 'growth rate' of work at that channel. This expression is minimized by following the permutation based on ordering the stations in *increasing* values of the index $\frac{a_i}{\alpha_i}$. Moreover, if a two-cycle horizon is considered, then, in expectation, the length of this horizon is minimized if the server performs the second Hamiltonian tour in the *reverse* order of the first tour. This implies an Elevator type polling scheme (see also Yechiali [1991]).

Elevator (or scan) type polling systems have been already studied in the literature. Coffman and Hofri [1982] analyzed the Exhaustive service regime assuming *constant* service and switch-over (seek) times. They derived the Probability-Generating-Functions (PGFs) of the number of customers (packets) at the various queues at polling instants, as well as at switch-over times. Expressions for customers' mean waiting times are then calculated. Swartz [1982] considered a slotted-time model under the Gated service discipline and obtained the PGF of the state of the system at polling instants. Takagi & Murata [1986] further analyzed scan-type TDM and polling systems under both the Exhaustive and Gated service regimes. They computed the mean delay values of requests at the various stations and show that a discrimination exists among the stations due to their relative positions. Models with a single buffer for each queue has also been studied for a cyclic (Hamiltonian) type polling scheme (Browne & Yechiali [1991]), as well as for heterogeneous scan polling procedure (Bunday, Sztrik & Tapsir [1992]). Altman, Khamisy & Yechiali [1992] introduced and studied the Elevator Globally-Gated scheme. A surprising result for that system is that mean waiting times at *all* stations are the same.

In this paper we employ a *unified* approach to study, analyze and extend results for four service regimes under the Elevator type polling scheme. The regimes are the Exhaustive, Gated, Globally-Quasi-Exhaustive and Globally-Gated. We derive a new and general formula for calculating mean waiting times of customers in various queues in an arbitrary polling scheme (not necessarily of Elevator-type), and use this formula for calculating performance measures for each of the service regimes mentioned above. Furthermore, we calculate the server's sojourn times in each queue in the 'up' and 'down' directions, separately, and find conditions under which the 'up' and 'down' cycles are equal.

For the Globally-Gated regime we reestablish the surprising result (Altman, Khamisy & Yechiali [1992]) that *all* mean waiting times are equal. This is the only known non-symmetric polling scheme that achieves such a 'fairness' phenomenon.

The structure of the paper is as follows: In Section 2 a general description of Elevator-type procedures (independent of the service discipline in each channel) is presented. In Section 3 we derive a *general* formula for calculating mean waiting times of customers in the various channels. In Sections 4, 5, 6 and 7 we provide analyses for the Elevator Exhaustive, Elevator Gated, Elevator-Quasi-Exhaustive and Elevator Globally-Gated systems, respectively.

2 The Model

We consider a polling system consisting of a single server and N independent infinite-buffer queues (channels). Customers arrive at queue i ($i = 1, 2, ..., N$) according to a Poisson process with rate λ_i. The server moves from one channel to another in an elevator (scan) fashion: it first serves the channels in one direction, i.e. in the order $1, 2, ..., N$ ('up' direction), and then reverses its orientation and serves the stations in the opposite ('down') direction, going through stations $N, N-1, ..., 1$. The server stays at channel i ($i = 1, 2, ..., N$) for a length of time determined by the service discipline and then moves to channel $i + 1$ or $i - 1$, according to the polling orientation. It keeps moving from one channel to another even when there are no customers in the system. Each customer in channel i carries an independent random service requirement distributed as V_i and having distribution function $G_i(.)$. The flow-rate of work to channel i is $\rho_i = \lambda_i E(V_i)$, and the total flow-rate of work into the system is $\rho = \sum_{i=1}^{N} \rho_i$.

When leaving channel i and before moving to the next channel the server incurs a switch-over (walking time) period, the duration of which is a random variable. This variable denotes the switch-over time from channel i to channel $i+1$ in the up direction, and from channel $i+1$ to channel i in the down direction (θ_i^{up} and θ_i^{down}, respectively). In various applications it is common to assume that all switch-over times are independent, and for each i the up and down switching distributions are the same, i.e. $\theta_i = \theta_i^{up} = \theta_i^{down}$. The period during which the server moves up (down) is called an 'up' ('down') cycle, and is denoted by C_1 (C_2). A full cycle is $C = C_1 + C_2$. Finally, throughout the paper the Laplace-Stieltjes-Transform (LST) of a random variable X is denoted by $\tilde{X}(\omega) = E\{\exp(-\omega X)\}$.

3 A General Result For Mean Waiting Times

A common method for calculating mean waiting times incurred by customers in the various queues in polling systems is presented in Takagi [1986], where specific calculations for the cyclic Exhaustive and the cyclic Gated regimes are performed. The method is based on obtaining a set of implicit equations for the PGFs of the number of customers found at the various channels at polling instants, differentiating these PGFs and deriving a set of linear equations whose solution enables one to calculate the desired mean waiting times.

We develop an alternative method for obtaining mean waiting times in an arbitrary polling system. The method is based on a derivation of a *general* equation for $E(W_i)$, the mean waiting time of customer in queue i, which is then used for each service regime according to its specific features. The mean waiting times in all Elevator-Type schemes mentioned above are derived with the aid of this equation by calculating the waiting time in the up cycle and in the down cycle, separately. The mean waiting time is then given by a weighted sum of the two means, where the weights are the probabilities of finding the server in the up cycle or in the down cycle, respectively.

Consider the probability generating function, $Q_i(z) = E(z^{L_i})$, of the number of customers, L_i, left behind by an arbitrary departing customer from channel i. As the distribution of number of customers in the system at epochs of arrival and epochs of departure are identical, then by the well known PASTA phenomenon (Poisson Arrivals See Time Averages), $Q_i(z)$ also stands for the generating function of the number of customers at channel i in a steady state regime at an arbitrary point of time.

Consider the system in steady-state. Let T_i be the total number of customers served in channel i during a visit of the server to that channel, and let $L_i(n) \quad (n = 1, 2, ..., T_i)$, be the sequence of random variables denoting the number of customers that the n-th departing customer from channel i (counting from the moment that the channel was last polled) leaves behind him. Then the PGF is given (see Takagi [1986], p. 78) by

$$Q_i(z) = \frac{E\left(\sum_{n=1}^{T_i} z^{L_i(n)}\right)}{E\left(T_i\right)} \tag{2}$$

Let X_i^i denote the number of customers present at channel i at its polling instant, and denote by $V_i(n)$ the total service time of n customers in channel i. Also let $A_i(t)$ be the number of Poisson arrivals to channel i during a time interval of length t. Then, $L_i(n) = X_i^i - n + A_i(V_i(n))$. Thus, the evaluation of the expression for $Q_i(z)$ becomes:

$$Q_i(z) = \frac{1}{E\left(T_i\right)} E\left(\sum_{n=1}^{T_i} z^{X_i^i - n + A_i(V_i(n))}\right) = \frac{1}{E\left(T_i\right)} E\left(z^{X_i^i} \sum_{n=1}^{T_i} z^{-n + A_i(V_i(n))}\right)$$

$$= \frac{1}{E\left(T_i\right)} E\left(z^{X_i^i} \sum_{n=1}^{T_i} z^{-n} e^{-\lambda_i V_i(n)(1-z)}\right) = \frac{1}{E\left(T_i\right)} E\left(z^{X_i^i} \sum_{n=1}^{T_i} \left[\frac{\tilde{V}_i(\lambda_i(1-z))}{z}\right]^n\right)$$

$$= \frac{1}{E\left(T_i\right)} E\left(z^{X_i^i} \times \frac{\tilde{V}_i(\lambda_i(1-z))}{z} \times \frac{1 - \left[\frac{\tilde{V}_i(\lambda_i(1-z))}{z}\right]^{T_i}}{1 - \frac{\tilde{V}_i(\lambda_i(1-z))}{z}}\right)$$

$$= \frac{1}{E\left(T_i\right)} \times \frac{1}{z - \tilde{V}_i(\lambda_i(1-z))} E\left[z^{X_i^i - T_i} \tilde{V}_i(\lambda_i(1-z)) \left(z^{T_i} - \left[\tilde{V}_i(\lambda_i(1-z))\right]^{T_i}\right)\right] \tag{3}$$

The average number of customers at channel i at an arbitrary point of time is given by:

$$E(L_i) = \left.\frac{\partial Q_i(z)}{\partial z}\right|_{z=1} = \frac{E(T_i^2) - E(T_i)}{2E(T_i)}(1 + \rho_i) + \frac{E(X_i^i T_i) - E(T_i^2)}{2E(T_i)} + \rho_i \tag{4}$$

Let W_i denote the waiting time of an arbitrary customer at queue i. The Laplace Stieltjes Transform (LST) of W_i and its expectation are obtained using the well known relations:

$$\tilde{W}_i(\lambda_i(1-z))\tilde{V}_i(\lambda_i(1-z)) = Q_i(z)$$

$$\lambda_i E(W_i) + \lambda_i E(V_i) = E(L_i) \tag{5}$$

Thus, the average waiting time for an arbitrary customer at channel i is given by

$$E(W_i) = \frac{E(T_i^2) - E(T_i)}{2\lambda_i E(T_i)}(1+\rho_i) + \frac{E(X_i^i T_i) - E(T_i^2)}{2\lambda_i E(T_i)} \tag{6}$$

The following theorem gives some insight into the result given by Eq. (6).

Theorem 1 *Let* $A_i = A_i(H_i + R_i)$ *and* $X_i = A_i(H_i)$ *be two Poisson random variables (each with intensity* λ_i*), representing the number of customers that have arrived to channel* i *during some random periods* $H_i + R_i$ *and* H_i*, respectively. Then*

$$\frac{E(A_i^2) - E(A_i)}{2E(A_i)} = \lambda_i \frac{E\left[(H_i + R_i)^2\right]}{2E\left(H_i + R_i\right)} \tag{7}$$

$$\frac{E(X_i A_i) - E(A_i^2)}{2E(A_i)} = -\frac{\lambda_i E(H_i R_i) + \lambda_i E(R_i^2) + E(R_i)}{2E(H_i + R_i)} \tag{8}$$

Proof: As A_i is Poisson, $E(A_i) = \lambda_i E(H_i + R_i)$

$$E(A_i^2) = E\left[(A_i(H_i + R_i))^2\right] = E_{H_i,R_i}\left[E\left[(A_i(H_i+R_i))^2\right]\right] =$$

$$E_{H_i,R_i}\left[\lambda_i^2 (H_i + R_i)^2 + \lambda_i(H_i + R_i)\right] = \lambda_i^2 E\left[(H_i + R_i)^2\right] + \lambda_i E\left[H_i + R_i\right]$$

Thus, $\frac{E(A_i^2) - E(A_i)}{2E(A_i)} = \lambda_i \frac{E\left[(H_i+R_i)^2\right]}{2E[H_i + R_i]}$. Also, as $A_i(H_i + R_i) = A_i(H_i) + A_i(R_i)$, we have

$$E(X_i A_i) = E_{H_i,R_i}\left\{E\left[(A_i(H_i))^2 + A_i(H_i)A_i(R_i)\right]\right\} = E_{H_i,R_i}\left[\lambda_i^2 H_i^2 + \lambda_i H_i + \lambda_i^2 H_i R_i\right]$$

$$= \lambda_i^2 E(H_i^2) + \lambda_i E(H_i) + \lambda_i^2 E(H_i R_i)$$

Thus,

$$\frac{E(X_i A_i) - E(A_i^2)}{2E(A_i)} = -\frac{\lambda_i E(H_i R_i) + \lambda_i E(R_i^2) + E(R_i)}{2E(H_i + R_i)}$$

■

Suppose that in some polling systems the number of customers present in channel i at its polling instant, X_i^i, is the number of Poisson arrivals to that channel during some random time H_i, i.e $X_i^i = A_i(H_i)$, and T_i, the total number of customers served during a visit to channel i is given by $A_i(H_i+R_i)$. Then, it follows from Theorem 1 that

$$E(W_i) = \frac{E\left[(H_i+R_i)^2\right]}{2E\left[H_i+R_i\right]}(1+\rho_i) - \frac{\lambda_i E(H_iR_i) + \lambda_i E(R_i^2) + E(R_i)}{2\lambda_i E\left[H_i+R_i\right]} \tag{9}$$

That is, the mean waiting time is comprised of three elements, the first two of which are

(i) The mean residual time of the random time period $H_i + R_i$ (given by $\alpha(H,R) \equiv \frac{E\left[(H_i+R_i)^2\right]}{2E[H_i+R_i]}$).

(ii) The service time required by customers who have arrived at that channel during the past part of the period $H_i + R_i$, but before the arrival of the specific customer (given by $\rho_i\alpha(H,R)$).

The last term in Eq. (9) is due to the dependence of the random variables X_i^i and T_i.

4 Elevator Exhaustive Scheme

In this section we analyze the Elevator-polling Exhaustive-service scheme. We calculate expressions for the LST and means of the sojourn times of the server in various queues in the up and down cycles, and derive explicit expressions for the mean durations of the up and down cycles. Finally, we obtain formulae for calculating mean waiting times of arbitrary customers in the various queues, both in the up and down directions.

4.1 Cycle Times

Suppose that at time 0 the state of the system is $(n_1, n_2, ..., n_N)$, where n_i is the number of customers present in channel i. The server than starts its up cycle serving each channel until it is empty, and then moves on to the next channel. Upon completion service at channel N the server starts its down movement (at that moment, because of the Exhaustive service discipline, channel N is empty). The down cycle is completed when channel 1 is exhausted. A new full cycle then starts again. It is thus clear that $n_1 = n_N = 0$ in every future cycle. That is, the server visits channels 1 and N only once in each full cycle, whereas channels $2, \ldots, N-1$ are each visited twice.

Let $Y_i^{(1)}$ and $Y_i^{(2)}$ be the occupation time of the server in channel i in the up and down directions, respectively. Then,

$$Y_i^{(1)} = \sum_{j=1}^{n_i} B_{ij} + \sum_{j=1}^{A_i(S_{i-1}^{(1)})} B_{ij} + \theta_i^{up}, \quad i = 1, 2, \ \ldots \ , N \tag{10}$$

where $\{B_{ij}\}_{j=1}^{\infty}$ is a sequence of i.i.d random variables all distributed as an $M/G_i/1$-type busy period with mean $E(B_i) = E(V_i)/(1-\rho_i)$, second moment $E(B_i^2) = E(V_i^2)/(1-\rho_i)^3$ and LST $\tilde{B}_i(\omega)$. $A_i(t)$ is a Poisson random variable with rate λ_i, counting the number of arrivals to channel i during a time period of length t. $S_{i-1}^{(1)} = \sum_{j=1}^{i-1} Y_j^{(1)}$ is the entrance time to channel i. The LST of $Y_i^{(1)}$ is thus given by

$$\tilde{Y}_i^{(1)}(\omega) = \left[\tilde{B}_i(\omega)\right]^{n_i} \tilde{S}_{i-1}^{(1)} \left(\lambda_i(1 - \tilde{B}_i(\omega))\right) \tilde{\theta}_i^{up}(\omega), \quad i = 1, 2, \ldots, N. \tag{11}$$

Eq. (11) leads to

$$E(Y_i^{(1)}) = \frac{n_i E(V_i)}{1-\rho_i} + \frac{\rho_i}{1-\rho_i} \sum_{j=1}^{i-1} E(Y_j^{(1)}) + E(\theta_i^{up}), \quad i = 1, 2, \ldots, N \tag{12}$$

where $\rho_i = \lambda_i E(V_i)$ is the amount of work flowing to channel i per unit time. Using the definition $Z_i^{(1)} = E(S_i^{(1)})$ and adding $Z_{i-1}^{(1)}$ to both sides of Eq. (12), we obtain a set of difference equations in $Z_i^{(1)}$,

$$Z_i^{(1)} - \frac{1}{1-\rho_i} Z_{i-1}^{(1)} = \frac{n_i E(V_i)}{1-\rho_i} + E(\theta_i^{up}), \quad i = 1, 2, \ldots, N \quad ; \ Z_0^{(1)} = 0. \tag{13}$$

The solution of Eq. (13) is

$$Z_i^{(1)} = \sum_{j=1}^{i} \left[\frac{n_j E(V_j) + (1-\rho_j) E(\theta_j^{up})}{1-\rho_j}\right] \prod_{r=j+1}^{i} (1 + \frac{\rho_r}{1-\rho_r}), \quad i = 1, 2, \ldots, N. \tag{14}$$

The mean duration of the up cycle is given by substituting $i = N$ in Eq. (14):

$$E(C_1) = Z_N^{(1)} = \sum_{j=1}^{N} \left[\frac{n_j E(V_j) + (1-\rho_j) E(\theta_j^{up})}{1-\rho_j} \right] \prod_{r=j+1}^{N} (1 + \frac{\rho_r}{1-\rho_r}), \qquad (15)$$

Observing the system at the end of the up cycle, the system's state is $(A_1(\theta_1^{up} + S_N^{(1)} - S_1^{(1)}),\ A_2(\theta_2^{up} + S_N^{(1)} - S_2^{(1)})\ ,\ .....,\ A_{N-1}(\theta_{N-1}^{up} + S_N^{(1)} - S_{N-1}^{(1)}),$ $0)$. The length of time required by the server to move back from channel N to channel i is $\sum_{r=0}^{N-i-1} Y_{N-r}^{(2)}$. Hence,

$$Y_i^{(2)} = \sum_{j=1}^{A_i\left(\theta_i^{up} + S_N^{(1)} - S_i^{(1)} + \sum_{r=0}^{N-i-1} Y_{N-r}^{(2)}\right)} B_{ij} \ + \ \theta_{i-1}^{down}$$

$$= \sum_{j=1}^{A_i\left(\theta_i^{up} + \sum_{r=i+1}^{N} (Y_r^{(1)} + Y_r^{(2)})\right)} B_{ij} \ + \ \theta_{i-1}^{down}, \quad i = 1, 2,\ ...\ , N \qquad (16)$$

with LST

$$\tilde{Y}_i^{(2)}(\omega) =$$

$$E\left\{ \exp\left[-\lambda_i (1 - \tilde{B}_i(\omega)) \sum_{r=i+1}^{N} (Y_r^{(1)} + Y_r^{(2)}) \right] \right\} \tilde{\theta}_i^{up}\left(\lambda_i (1 - \tilde{B}_i(\omega))\right) \tilde{\theta}_{i-1}^{down}(\omega),$$
$$i = 1, 2, \ldots, N. \quad (17)$$

The mean value of $Y_i^{(2)}$ is

$$E(Y_i^{(2)}) = \lambda_i E(B_i) \left[\sum_{r=i+1}^{N} \left(E(Y_r^{(1)}) + E(Y_r^{(2)}) \right) + E(\theta_i^{up}) \right] + E(\theta_{i-1}^{down}),$$
$$i = 1, 2, \ldots, N. \qquad (18)$$

By adding $Z_{i+1}^{(2)} = \sum_{r=i+1}^{N} E(Y_r^{(2)})$ to both sides of Eq. (18) we obtain a system of difference equations

$$Z_i^{(2)} - \frac{1}{1-\rho_i} Z_{i+1}^{(2)} = \frac{\rho_i}{1-\rho_i} \left[\sum_{r=i+1}^{N} E(Y_r^{(1)}) + E(\theta_i^{up}) \right] + E(\theta_{i-1}^{down}),$$

$$i = 1, 2, \ldots, N \quad ; \; Z_N^{(2)} = 0 \quad (19)$$

whose solution is

$$Z_i^{(2)} = \sum_{j=i}^{N} \left[\frac{\rho_j \left[\sum_{r=j+1}^{N} E(Y_r^{(1)}) + E(\theta_j^{up}) \right] + (1-\rho_j) E(\theta_{j-1}^{down})}{1-\rho_j} \right] \prod_{r=i}^{j-1} (1 + \frac{\rho_r}{1-\rho_r}),$$

$$i = 1, 2, \ldots, N. \quad (20)$$

Now, the mean down cycle time is given by substituting $i = 1$ in Eq. (20):

$$E(C_2) = Z_1^{(2)} =$$

$$\sum_{j=1}^{N} \left[\frac{\rho_j \left[\sum_{r=j+1}^{N} E(Y_r^{(1)}) + E(\theta_j^{up}) \right] + (1-\rho_j) E(\theta_{j-1}^{down})}{1-\rho_j} \right] \prod_{r=1}^{j-1} (1 + \frac{\rho_r}{1-\rho_r}) \quad (21)$$

As $E(Y_r^{(1)}) = Z_r^{(1)} - Z_{r-1}^{(1)}$, Eq. (14) and Eq. (20) determine explicitly the $2N$ values of $Z_r^{(1)}$ and $Z_r^{(2)}$ ($r = 1, 2, ..., N$) for any initial system-state $(n_1, n_2, ..., n_N)$. The mean cycle time is given by the sum of the mean up cycle and the mean down cycle (Eq. (15) and Eq. (21) respectively).

In general the *expected* value of $n_j E(V_j)$ is $\rho_j [\sum_{r=1}^{j-1} E(Y_r^{(2)}) + E(\theta_{j-1}^{down})]$, which is the expected amount of work flowing into channel j from the moment the server leaves the channel in the down direction until it reenters it in its up direction. Substituting the above expression for $n_j E(V_j)$ in Eq. (15), we express $E(C_1)$ in terms of $E(Y_r^{(2)})$, similarly to Eq. (21). In order to find expressions for $Y_i^{(1)}$ and $Y_i^{(2)}$ for an *arbitrary* cycle, we note that the number of customers present in queue i at the beginning of an up cycle is $n_i = A_i \left(\theta_{i-1}^{down} + \sum_{r=1}^{i-1} Y_r^{(2)} \right)$. Thus Eq. (10) is transformed into

$$Y_i^{(1)} = \sum_{j=1}^{A_i \left(\theta_{i-1}^{down} + \sum_{r=1}^{i-1} (Y_r^{(1)} + Y_r^{(2)}) \right)} B_{ij} \; + \; \theta_i^{up}, \quad i = 1, 2, ..., N \qquad (22)$$

with LST

$$\tilde{Y}_i^{(1)}(\omega) =$$

$$E \left\{ \exp \left[-\lambda_i (1 - \tilde{B}_i(\omega)) \sum_{r=1}^{i-1} (Y_r^{(1)} + Y_r^{(2)}) \right] \right\} \tilde{\theta}_{i-1}^{down} \left(\lambda_i (1 - \tilde{B}_i(\omega)) \right) \tilde{\theta}_i^{up}(\omega),$$

$$i = 1, 2, \ldots, N. \quad (23)$$

Then

$$E(Y_i^{(1)}) = \lambda_i E(B_i) \left[\sum_{r=1}^{i-1} \left(E(Y_r^{(1)}) + E(Y_r^{(2)}) \right) + E(\theta_{i-1}^{down}) \right] + E(\theta_i^{up}),$$

$$i = 1, 2, \ldots, N. \quad (24)$$

The server's total mean occupation time at channel i during a full cycle is

$$E(Y_i^{(1)}) + E(Y_i^{(2)}) =$$

$$\frac{\rho_i}{1-\rho_i} \left[\sum_{j=1, j\neq i}^{N} \left(E(Y_j^{(1)}) + E(Y_j^{(2)}) \right) + E(\theta_{i-1}^{down}) + E(\theta_i^{up}) \right] + E(\theta_{i-1}^{down}) + E(\theta_i^{up}),$$

$$i = 1, 2, \ldots, N. \quad (25)$$

It follows that

$$E(Y_i^{(1)}) + E(Y_i^{(2)}) = \rho_i E(C) + E(\theta_{i-1}^{down}) + E(\theta_i^{up}), \quad i = 1, 2, \ldots, N. \quad (26)$$

Substituting Eq. (26) in Eq. (24) yields

$$E(Y_i^{(1)}) =$$

$$\frac{\rho_i}{1-\rho_i} \left[\sum_{r=1}^{i-1} \left(E(\theta_{r-1}^{down}) + E(\theta_r^{up}) + \rho_r E(C) \right) + E(\theta_{i-1}^{down}) \right] + E(\theta_i^{up}), \quad i = 1, 2, \ldots, N. \quad (27)$$

In a similar manner, using Eq. (18),

$$E(Y_i^{(2)}) =$$

$$\frac{\rho_i}{1-\rho_i} \left[\sum_{r=i+1}^{N} \left(E(\theta_r^{up}) + E(\theta_{r-1}^{down}) + \rho_r E(C) \right) + E(\theta_i^{up}) \right] + E(\theta_{i-1}^{down}), \quad i = 1, 2, \ldots, N. \quad (28)$$

Clearly the mean up (down) cycle, $E(C_1)$ ($E(C_2)$), is given by summing the expressions for $E(Y_i^{(1)})$ $(E(Y_i^{(2)}))$. The total mean cycle time is derived by summing the expressions from Eq. (26), and is given, as expected, by

$$E(C) = \frac{\sum_{i=1}^{N}\left[E(\theta_{i-1}^{down}) + E(\theta_i^{up})\right]}{1 - \sum_{i=1}^{N}\rho_i} \tag{29}$$

(See Watson [1984] for a general result, and Takagi & Murata [1986] for a slotted model).

4.2 Comparison between the up and down cycles

We are interested in finding conditions under which the means of the up and the down cycles are equal. We have

Proposition 4.1 *In the Elevator-Exhaustive model,* $E(C_1) = E(C_2)$ *whenever* $\theta_i^{up} = \theta_{N-i}^{down}$ *and* $\rho_i = \rho_{N-i+1}$ $(i = 1, \ldots, N)$.

Proof: Using Eq. (27) and Eq. (28)

$$E(C_1) - E(C_2) = \sum_{i=1}^{N}\left(E(Y_i^{(1)}) - E(Y_i^{(2)})\right)$$

$$= \sum_{i=1}^{N}\frac{\rho_i}{1-\rho_i}\left[\sum_{r=1}^{i-1}\left(E(\theta_{r-1}^{down}) + E(\theta_r^{up}) + \rho_r E(C)\right) + E(\theta_{i-1}^{down})\right]$$

$$-\sum_{i=1}^{N}\frac{\rho_i}{1-\rho_i}\left[\sum_{r=i+1}^{N}\left(E(\theta_r^{up}) + E(\theta_{r-1}^{down}) + \rho_r E(C)\right) + E(\theta_i^{up})\right] \tag{30}$$

Substituting $\theta_i^{up} = \theta_{N-i}^{down}$ and $\rho_i = \rho_{N-i+1}$ for $i = 1, \ldots, N$, in the second term of Eq. (30), setting $k = N - i + 1$, and using $\sum_{i=1}^{N}\rho_i E(\theta_i^{up})/(1-\rho_i) = \sum_{i=1}^{N}\rho_i E(\theta_i^{down})/(1-\rho_i)$, we get

$$E(C_1) - E(C_2) = \sum_{i=1}^{N}\frac{\rho_i}{1-\rho_i}\left[\sum_{r=1}^{i-1}\left(E(\theta_{r-1}^{down}) + E(\theta_r^{up}) + \rho_r E(C)\right) + E(\theta_{i-1}^{down})\right]$$

$$-\sum_{i=1}^{N}\frac{\rho_{N-i+1}}{1-\rho_{N-i+1}}\left[\sum_{k=1}^{N-i}\left(E(\theta_{k-1}^{down}) + E(\theta_k^{up}) + \rho_k E(C)\right) + E(\theta_{i-1}^{down})\right]$$

Substituting $j = N - i + 1$, we readily have $E(C_1) = E(C_2)$. ■

4.3 Mean Waiting Times

The mean waiting time in the Elevator-Exhaustive system can be calculated utilizing Eq. (6) derived in Section 3. Clearly, the total mean waiting time in channel i is given by a weighted sum of the mean waiting times in the up cycle, $E(W_i|up)$, and the down cycle, $E(W_i|down)$. Therefore,

$$E(W_i) = \frac{1}{E(C)}\left[E(C_1)E(W_i|up) + E(C_2)E(W_i|down)\right] \tag{31}$$

To calculate $E(W_i|up)$ we recall that the number of customers found by the server at channel i at polling instant of that channel in the up direction, $X_i^i(up)$, is the number of customers that have arrived at channel i during the time interval starting from the moment the server finished serving channel i in the down direction, until it first returns to that channel in the up direction. That is,

$$X_i^i(up) = A_i\left(\theta_{i-1}^{down} + \sum_{r=1}^{i-1}\left(Y_r^{(2)} + Y_r^{(1)}\right)\right)$$

The number of customers served at channel i in the up direction, denoted by $T_i(up)$, is given by

$$T_i(up) = X_i^i(up) + A_i\left(\sum_{j=1}^{X_i^i(up)} B_{ij}\right)$$

Thus, the waiting time in the up direction in the Elevator-Exhaustive system is given by Eq. (6) with $X_i^i(up)$ and $T_i(up)$ replacing X_i^i and T_i, respectively.

In a similar manner, denoting by $X_i^i(down)$, the number of customers found by the server at channel i when polled in the down direction, and by $T_i(down)$ the number of customers served at channel i in the down direction, we have

$$X_i^i(down) = A_i\left(\theta_i^{up} + \sum_{r=i+1}^{N}\left(Y_r^{(1)} + Y_r^{(2)}\right)\right)$$

$$T_i(down) = X_i^i(down) + A_i\left(\sum_{j=1}^{X_i^i(down)} B_{ij}\right)$$

Again, the waiting time in the down direction is given by substituting $X_i^i(down)$ and $T_i(down)$ in Eq.(6).

Using X_i^i and T_i instead of $X_i^i(up)$ and $T_i(up)$, respectively, we have

$$E(T_i) = E\left(X_i^i + A_i\left(\sum_{j=1}^{X_i^i} B_{ij}\right)\right) = E(X_i^i) + \lambda_i E(B_i)E(X_i^i) = \frac{E(X_i^i)}{1-\rho_i}$$

$$E(T_i^2) = E\left[\left(X_i^i + A_i\left(\sum_{j=1}^{X_i^i} B_{ij}\right)\right)^2\right]$$

$$= E\left[(X_i^i)^2\right] + 2\frac{\rho_i E\left[(X_i^i)^2\right]}{1-\rho_i} + \lambda_i^2 E(X_i^i)Var(B_i) + E\left[(X_i^i)^2\right]\lambda_i^2[E(B_i)]^2 + \frac{\rho_i E(X_i^i)}{1-\rho_i}$$

$$= E\left[(X_i^i)^2\right]\left(\frac{\rho_i}{1-\rho_i}\right)^2 + E(X_i^i)\left[\lambda_i^2 Var(B_i) + \frac{\rho_i}{1-\rho_i}\right]$$

and

$$E(X_i^i T_i) = E\left(X_i^i\left(X_i^i + A_i\left(\sum_{j=1}^{X_i^i} B_{ij}\right)\right)\right) = E\left[(X_i^i)^2\right] + \lambda_i E(B_i)E\left[(X_i^i)^2\right] = \frac{E\left[(X_i^i)^2\right]}{1-\rho_i}$$

Substituting in Eq. (6), we derive,

$$E(W_i|up) = \frac{E\left[(X_i^i(up))^2\right]\lambda_i^2(E(B_i))^2 + E(X_i^i(up))\left[\lambda_i^2 Var(B_i) - 1\right]}{2\lambda_i E(X_i^i(up))}(1-\rho_i^2)$$
$$+\frac{E\left[(X_i^i(up))^2\right] - E(X_i^i(up))}{2\lambda_i E(X_i^i(up))}. \qquad (32)$$

$E(W_i|down)$ is obtained similarly with $X_i^i(down)$ replacing $X_i^i(up)$ in Eq. (32).

In order to complete the evaluation of $E(W_i)$ (as given by Eq.(31)), it is just left to calculate $E(X_i^i)$ and $E\left[(X_i^i)^2\right]$.

4.4 Generating Functions

The values of the first and the second moments of $X_i^i(up)$ and $X_i^i(down)$ will now be derived from a set of N PGFs as follows. In a similar way to the method used in Takagi [1986], we define $X_i^j(up)$ and $X_i^j(down)$ as the number of customers at channel j at polling instant of channel i at the up and down cycle, respectively. Recall that during a full cycle the server visits each of the channels $2, 3, \ldots, N-1$, twice (once in each direction), where channels 1 and N are visited only once. Let $F_i^{up}(\underline{z})$ and $F_i^{down}(\underline{z})$ be the generating functions describing the state-vector of the system at the polling instant of channel i in the up and down directions, respectively. That is

$$F_i^{up}(\underline{z}) \stackrel{\text{def}}{=} E\left[\prod_{j=1}^{N} z_j^{X_i^j(up)}\right], \quad i = 2, \ldots, N$$

$$F_i^{down}(\underline{z}) \stackrel{\text{def}}{=} E\left[\prod_{j=1}^{N} z_j^{X_i^j(down)}\right], \quad i = 1, 2, \ldots, N-1. \tag{33}$$

The evolution of the state of the system in the up direction is described by

$$X_{i+1}^j(up) = \begin{cases} X_i^j(up) + A_j(\theta_i^{up}) + A_j\left(\sum_{j=1}^{X_i^i(up)} B_{ij}\right) & j \neq i \\ \\ A_i(\theta_i^{up}) & j = i \end{cases} \tag{34}$$

Thus,

$$F_{i+1}^{up}(\underline{z}) = E\left[\prod_{j=1}^{N} z_j^{X_{i+1}^j(up)}\right]$$

$$= \tilde{\theta}_i^{up}\left(\sum_{j=1}^{N} \lambda_j(1-z_j)\right) F_i^{up}\left(z_1, ..., z_{i-1}, \tilde{B}_i\left(\sum_{j=1, j\neq i}^{N} \lambda_j(1-z_j)\right), z_{i+1}, ..., z_N\right),$$

$$i = 2, \ldots, N-1. \tag{35}$$

For $i = 1$

$$X_2^j(up) = \begin{cases} X_1^j(down) + A_j(\theta_1^{up}) + A_j\left(\sum_{j=1}^{X_1^1(down)} B_{1j}\right) & j \geq 2 \\ \\ A_1(\theta_1^{up}) & j = 1 \end{cases} \tag{36}$$

Then,

$$F_2^{up}(\underline{z}) = \tilde{\theta}_1^{up}\left(\sum_{j=1}^{N} \lambda_j(1 - z_j)\right) F_1^{down}\left(\tilde{B}_1\left(\sum_{j=1, j\neq 1}^{N} \lambda_j(1 - z_j)\right), z_2, ..., z_N\right). \tag{37}$$

Note that there is no expression for $F_1^{(up)}(\underline{z})$, as there is no polling of channel 1 in the up direction. The evolution of the state of the system in the down direction is

$$X_{i-1}^j(down) = \begin{cases} X_i^j(down) + A_j(\theta_{i-1}^{down}) + A_j\left(\sum_{j=1}^{X_i^i(down)} B_{ij}\right) & j \neq i \\ \\ A_i(\theta_{i-1}^{down}) & j = i \end{cases} \tag{38}$$

$$X_{N-1}^j(down) = \begin{cases} X_N^j(up) + A_j(\theta_{N-1}^{down}) + A_j\left(\sum_{j=1}^{X_N^N(up)} B_{Nj}\right) & j \leq N - 1 \\ \\ A_N(\theta_{N-1}^{down}) & j = N \end{cases} \tag{39}$$

It follows that

$$F_{i-1}^{down}(\underline{z}) = E\left[\prod_{j=1}^{N} z_j^{X_{i-1}^j(down)}\right]$$

$$= \tilde{\theta}_{i-1}^{down}\left(\sum_{j=1}^{N} \lambda_j(1 - z_j)\right) F_i^{down}\left(z_1, ..., z_{i-1}, \tilde{B}_i\left(\sum_{j=1, j\neq i}^{N} \lambda_j(1 - z_j)\right), z_{i+1}, ..., z_N\right),$$

$$i = 2, \ldots, N - 1. \tag{40}$$

$$F_{N-1}^{down}(\underline{z}) = \tilde{\theta}_{N-1}^{down}\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right)F_N^{up}\left(z_1, z_2, ..., z_{N-1}, \tilde{B}_N\left(\sum_{j=1}^{N-1}\lambda_j(1-z_j)\right)\right) \tag{41}$$

Results for the corresponding slotted model where obtained by Takagi & Murata [1986] without specific attention to the end points of the cycle (channels 1 and N).

Let $f_i^{up}(j) = \left.\frac{\partial F_i^{up}(\underline{z})}{\partial z_j}\right|_{\underline{z}=1}$ and $f_i^{down}(j) = \left.\frac{\partial F_i^{down}(\underline{z})}{\partial z_j}\right|_{\underline{z}=1}$.

Clearly, $E(X_i^j(up)) = f_i^{up}(j)$ and $E(X_i^j(down)) = f_i^{down}(j)$.
Taking derivatives, we obtain a set of $2N(N-1)$ equations in the $2N(N-1)$ unknowns $f_i^{up}(j)$ and $f_i^{down}(j)$:

$$\begin{aligned}
f_{i+1}^{up}(j) &= f_i^{up}(j)+\lambda_j\left[E(\theta_i^{up}) + f_i^{up}(i)E(B_i)\right], \quad 2 \le i \le N-1,\ i \ne j \\
f_{j+1}^{up}(j) &= \lambda_j E(\theta_j^{up}) \\
f_2^{up}(j) &= f_1^{down}(j)+\lambda_j\left[E(\theta_1^{up}) + f_1^{down}(1)E(B_1)\right]
\end{aligned} \tag{42}$$

$$\begin{aligned}
f_{i-1}^{down}(j) &= f_i^{down}(j)+\lambda_j\left[E(\theta_{i-1}^{down}) + f_i^{down}(i)E(B_i)\right], \quad 2 \le i \le N-1,\ i \ne j \\
f_{j-1}^{down}(j) &= \lambda_j E(\theta_{j-1}^{down}) \\
f_{N-1}^{down}(j) &= f_N^{up}(j)+\lambda_j\left[E(\theta_{N-1}^{down}) + f_N^{up}(N)E(B_N)\right]
\end{aligned} \tag{43}$$

¿From Eqs. (42) and (43) it follows, as expected, that $f_j^{up}(j) + f_j^{down}(j) = \lambda_j(1-\rho_j)E(C)$. That is, the total number of customers served at channel j during a full cycle is equal to the number of customers arriving at that channel while the server is away.

Let $f_i^{up}(j,k) = \left.\frac{\partial^2 F_i^{up}(\underline{z})}{\partial z_j \partial z_k}\right|_{\underline{z}=1}$ and $f_i^{down}(j,k) = \left.\frac{\partial^2 F_i^{down}(\underline{z})}{\partial z_j \partial z_k}\right|_{\underline{z}=1}$. Then the second moments of $X_i^j(up)$ and $X_i^j(down)$ are given by
$E\left[X_i^i(up)^2\right] = f_i^{up}(i,i) + f_i^{up}(i)$
$E\left[X_i^i(down)^2\right] = f_i^{down}(i,i) + f_i^{down}(i)$.
$f_i^{up}(i,i)$ and $f_i^{down}(i,i)$ are calculated by solving $2N^2(N-1)$ equations in the $2N^2(N-1)$ unknowns $f_i^{up}(j,k)$ and $f_i^{up}(j,k)$. For $i = 2, \ldots, N-1$, we use Eq. (35) to get (with some abuse of notation)

$$\left.\frac{\partial^2 F_{i+1}^{up}(\underline{z})}{\partial z_j \partial z_k}\right|_{\underline{z}=1} = \left[\frac{\partial^2 \tilde{\theta}_i^{up}}{\partial z_j \partial z_k}F_i^{up} + \frac{\partial \tilde{\theta}_i^{up}}{\partial z_k} \times \frac{\partial F_i^{up}}{\partial z_j} + \frac{\partial \tilde{\theta}_i^{up}}{\partial z_j} \times \frac{\partial F_i^{up}}{\partial z_k} + \tilde{\theta}_i^{up}\frac{\partial^2 F_i^{up}}{\partial z_j \partial z_k}\right]_{\underline{z}=1}$$

where

$$\frac{\partial^2 \tilde{\theta}_i^{up}}{\partial z_j \partial z_k} = \begin{cases} \lambda_j \lambda_k E((\theta_i^{up})^2) & j \neq k \\ \\ \lambda_j^2 E((\theta_i^{up})^2) & j = k \end{cases}$$

$$\frac{\partial^2 F_i^{up}(\underline{z})}{\partial z_j \partial z_k} = \begin{cases} f_i^{up}(i)\lambda_j\lambda_k E(B_i^2) + \lambda_k f_i^{up}(i,j)E(B_i) \\ +\lambda_j f_i^{up}(i,k)E(B_i) + f_i^{up}(j,k) + \\ f_i^{up}(i,i)\lambda_j\lambda_k (E(B_i))^2 & i \neq j \neq k \\ \\ f_i^{up}(i)\lambda_j^2 E(B_i^2) + 2\lambda_j f_i^{up}(i,j)E(B_i) + \\ f_i^{up}(j,j) + f_i^{up}(i,i)\lambda_j^2 (E(B_i))^2 & i \neq j = k \\ \\ 0 & i = j,\ i = k \end{cases}$$

$$\frac{\partial \tilde{\theta}_i^{up}}{\partial z_k} \times \frac{\partial F_i^{up}}{\partial z_j} = \lambda_k E(\theta_i^{up}) f_i^{up}(j)$$

$$\frac{\partial \tilde{\theta}_i^{up}}{\partial z_j} \times \frac{\partial F_i^{up}}{\partial z_k} = \lambda_j E(\theta_i^{up}) f_i^{up}(k)$$

Similar equations are derived from Eq. (37) using $F_2^{up}(\underline{z})$. In the same manner we use Eqs. (40) and (41) to derive the corresponding set of equations for $f_i^{down}(j,k)$.

Finally, by substituting the desired expressions for $E(X_i^i(up))$, $E\left[X_i^i(up)^2\right]$ in Eq.(32) one gets the value of $E(W_i|up)$. The corresponding expression for $E(W_i|down)$ is obtained similarly. Substituting these two terms in Eq. (31) yields the desired result for $E(W_i)$.

5 Elevator Gated Scheme

In this section we analyze the Elevator-polling Gated-service scheme. We calculate expressions for the LST and means of the sojourn times of the server in each channel during the up and down cycles, separately, and derive explicit expressions for the mean duration of a full cycle. Finally, we obtain formulae for calculating mean waiting times of customers in the various queues, both in the up and down directions, as well as in general.

5.1 Cycle Times

The analysis of the Elevator-Gated system requires only a slight modification of the corresponding analysis of the Elevator-Exhaustive scheme presented in Section 4. Under the Elevator-Gated scheme the server moves upwards along stations $1, 2, ..., N$, serving in each channel only those customers present upon entrance, and then moves in the opposite direction through stations $N, N-1, ..., 1$. In contrast with the Elevator-Exhaustive model, the server may leave unserved customers behind when it exits a channel. Thus, the server visits all channels (including channels 1 and N) twice in every full cycle.

Let $\{V_{ij}\}_{j=1}^{\infty}$ denote a sequence of independent random service requirements in channel i having a common distribution function $G_i(.)$ and LST $\tilde{V}_i(\omega)$. Let $Y_i^{(1)}$ and $Y_i^{(2)}$ denote the total occupation time of the server in channel i (from the moment that service begins until the end of the switch-over to the next channel) in up and down directions, respectively. Suppose that at the start of an up cycle the state of the system is $(n_1, n_2, ..., n_N)$. Then $Y_i^{(1)}$ is given by

$$Y_i^{(1)} = \sum_{j=1}^{n_i} V_{ij} + \sum_{j=1}^{A_i(S_{i-1}^{(1)})} V_{ij} + \theta_i^{up}, \quad i = 1, 2, \ ... \ , N, \tag{44}$$

with LST

$$\tilde{Y}_i^{(1)}(\omega) = \left[\tilde{V}_i(\omega)\right]^{n_i} \tilde{S}_{i-1}^{(1)}\left(\lambda_i(1-\tilde{V}_i(\omega))\right)\tilde{\theta}_i^{up}(\omega), \quad i = 1, 2, \ldots, N. \tag{45}$$

where $S_i^{(1)} = \sum_{j=1}^{i} Y_j^{(1)}$. The mean value of $Y_i^{(1)}$ is given by

$$E(Y_i^{(1)}) = n_i E(V_i) + \rho_i E(S_{i-1}^{(1)}) + E(\theta_i^{up}), \quad i = 1, 2, \ldots, N. \tag{46}$$

Using the definition $Z_i^{(1)} = E(S_i^{(1)})$ and adding $Z_{i-1}^{(1)}$ to both sides of Eq. (46), we obtain a set of difference equations in $\left\{Z_i^{(1)}\right\}$, i.e.,

$$Z_i^{(1)} - (1+\rho_i)Z_{i-1}^{(1)} = n_i E(V_i) + E(\theta_i^{up}), \quad i = 1, 2, \ldots, N \quad ; \ Z_0^{(1)} = 0 \tag{47}$$

The system (47) yields the solution

$$Z_i^{(1)} = \sum_{j=1}^{i} [n_j E(V_j) + E(\theta_i^{up})] \prod_{r=j+1}^{i} (1+\rho_r), \quad i = 1, 2, \ldots, N. \tag{48}$$

Thus, the mean value of the up cycle is

$$E(C_1) = Z_N^{(1)} = \sum_{j=1}^{N} [n_j E(V_j) + E(\theta_i^{up})] \prod_{r=j+1}^{N} (1+\rho_r). \tag{49}$$

Now,

$$Y_i^{(2)} = \sum_{j=1}^{A_i\left(Y_i^{(1)}+\sum_{r=i+1}^{N}(Y_r^{(1)}+Y_r^{(2)})\right)} V_{ij} \; + \; \theta_{i-1}^{down}, \quad i=1,2,\ \dots\ ,N \tag{50}$$

with LST

$$\tilde{Y}_i^{(2)}(\omega) = E\left\{\exp\left[-\lambda_i\left(1-\tilde{V}_i(\omega)\right)\left(Y_i^{(1)} + \sum_{r=i+1}^{N}\left(Y_r^{(1)}+Y_r^{(2)}\right)\right)\right]\right\}\tilde{\theta}_{i-1}^{down}(\omega), \quad i=1,2,\dots,N. \tag{51}$$

And mean value

$$E(Y_i^{(2)}) = \lambda_i E(V_i)\left[E(Y_i^{(1)}) + \sum_{r=i+1}^{N}\left(E(Y_r^{(1)})+E(Y_r^{(2)})\right)\right] + E(\theta_{i-1}^{down}), \quad i=1,2,\dots,N. \tag{52}$$

This leads to

$$Z_i^{(2)} - (1+\rho_i)Z_{i+1}^{(2)} = \rho_i \sum_{r=i}^{N} E(Y_r^{(1)}) + E(\theta_{i-1}^{(down)}), \quad i=1,2,\dots,N \;\; ; \; Z_N^{(2)} = 0 \tag{53}$$

with solution

$$Z_i^{(2)} = \sum_{j=i}^{N}\left[\rho_j\left(Z_N^{(1)} - Z_{j-1}^{(1)}\right) + E(\theta_{j-1}^{down})\right]\prod_{r=i}^{j-1}(1+\rho_r), \quad i=1,2,\dots,N. \tag{54}$$

The mean value for the down cycle is

$$E(C_2) = Z_1^{(2)} = \sum_{j=1}^{N}\left[\rho_j\left(Z_N^{(1)} - Z_{j-1}^{(1)}\right) + E(\theta_{j-1}^{down})\right]\prod_{r=1}^{j-1}(1+\rho_r), \tag{55}$$

as if $n_j^{(2)} = \lambda_j \left(Z_N^{(1)} - Z_{j-1}^{(1)} \right)$ is the number of customers at channel j at the end of the up cycle.

In order to derive explicit expressions for $Y_i^{(1)}$ and $Y_i^{(2)}$, we note that n_i, the number of customers present in queue i at the beginning of an up cycle, is the number of customers that have arrived to channel i during the time interval starting ¿From the moment the server has last entered channel i in its down direction, until the end of the entire down cycle. Therefore, $n_i = A_i \left(\sum_{r=1}^{i} Y_r^{(2)} \right)$. Then,

$$Y_i^{(1)} = \sum_{j=1}^{A_i\left(Y_i^{(2)} + \sum_{r=1}^{i-1}(Y_r^{(2)} + Y_r^{(1)})\right)} V_{ij} \ + \ \theta_i^{up} \quad (i = 1, 2, \ \dots \ , N) \tag{56}$$

with LST

$$\tilde{Y}_i^{(1)}(\omega) = E\left\{ \exp\left[-\lambda_i \left(1 - \tilde{V}_i(\omega)\right) \left(Y_i^{(2)} + \sum_{r=1}^{i-1} \left(Y_r^{(2)} + Y_r^{(1)} \right) \right) \right] \right\} \tilde{\theta}_i^{up}(\omega), \quad i = 1, 2, \dots, N. \tag{57}$$

And mean value

$$E(Y_i^{(1)}) = \lambda_i E(V_i) \left[E(Y_i^{(2)}) + \sum_{r=1}^{i-1} \left(E(Y_r^{(2)}) + E(Y_r^{(1)}) \right) \right] + E(\theta_i^{up}), \quad i = 1, 2, \dots, N. \tag{58}$$

The total mean occupation time of the server at channel i is

$$\begin{aligned} E(Y_i^{(1)}) &+ E(Y_i^{(2)}) \\ &= \rho_i \left[\sum_{j=1, j \neq i}^{N} \left(E(Y_j^{(1)}) + E(Y_j^{(2)}) \right) + E(Y_i^{(1)}) + E(Y_i^{(2)}) \right] + E(\theta_{i-1}^{down}) + E(\theta_i^{up}) \\ &= \rho_i E(C) + E(\theta_{i-1}^{down}) + E(\theta_i^{up}), \qquad i = 1, 2, \dots, N. \end{aligned} \tag{59}$$

Substituting the expression for $E(Y_i^{(1)}) + E(Y_i^{(2)})$ in Eq. (58) yields

$$E(Y_i^{(1)}) = \rho_i \left[\sum_{r=1}^{i-1} \left(E(\theta_{r-1}^{down}) + E(\theta_r^{up}) + \rho_r E(C) \right) + E(Y_i^{(2)}) \right] + E(\theta_i^{up}),$$

$$i = 1, 2, \ldots, N. \tag{60}$$

The same applies for $Y_i^{(2)}$ (using Eq. (52)),

$$E(Y_i^{(2)}) = \rho_i \left[\sum_{r=i+1}^{N} \Big(E(\theta_{r-1}^{down}) + E(\theta_r^{up}) + \rho_r E(C) \Big) + E(Y_i^{(1)}) \right] + E(\theta_{i-1}^{down}),$$

$$i = 1, 2, \ldots, N. \tag{61}$$

Eqs. (60) and (61) yield a set of $2N$ equations in the $2N$ unknowns $\left\{ Y_i^{(1)}, Y_i^{(2)} \right\}$, whose solution is given by

$$E(Y_i^{(1)}) = \frac{1}{1-\rho_i^2} \Bigg\{ E(\theta_i^{up}) + \rho_i \left[\sum_{r=1}^{i-1} \Big[E(\theta_{r-1}^{down}) + E(\theta_r^{up}) + \rho_r E(C) \Big] \right.$$

$$\left. + \rho_i \left[\sum_{r=i+1}^{N} \Big[E(\theta_r^{up}) + E(\theta_{r-1}^{down}) + \rho_r E(C) \Big] \right] + E(\theta_{i-1}^{down}) \right] \Bigg\} \tag{62}$$

$$E(Y_i^{(2)}) = \frac{1}{1-\rho_i^2} \Bigg\{ E(\theta_{i-1}^{down}) + \rho_i \left[\sum_{r=i}^{N-1} \Big[E(\theta_{r-1}^{down}) + E(\theta_r^{up}) + \rho_r E(C) \Big] \right.$$

$$\left. + \rho_i \left[\sum_{r=1}^{i-1} \Big[E(\theta_r^{up}) + E(\theta_{r-1}^{down}) + \rho_r E(C) \Big] \right] + E(\theta_i^{up}) \right] \Bigg\} \tag{63}$$

The mean total cycle time, is derived by summing over the occupation times in Eq.(59) and again, as expected, is

$$E(C) = \frac{\sum_{i=1}^{N} \Big[E(\theta_{i-1}^{down}) + E(\theta_i^{up}) \Big]}{1 - \sum_{i=1}^{N} \rho_i} \tag{64}$$

The mean up and down cycles are now calculated by setting $E(C_1) = \sum_{i=1}^{N} E(Y_i^{(1)})$ and $E(C_2) = \sum_{i=1}^{N} E(Y_i^{(2)})$.

5.2 Comparison between the up and down cycles

Similarly to the result obtained for the Elevator-Exhaustive case (Proposition 4.1), we have,

Proposition 5.1 *In the Elevator-Gated model, $E(C_1) = E(C_2)$ whenever $\theta_i^{up} = \theta_{N-i}^{down}$ and $\rho_i^{up} = \rho_{N-i+1}^{down}$ $(i = 1, \ldots, N)$.*

5.3 Mean Waiting Times

In any Gated service regime, the number of customers served during a visit of the server equals the number of customers present at the channel upon arrival. That is $X_i^i = T_i$. Thus, employing Eq. (6) for the classical (cyclic) Gated regime we have

$$E(W_i|Cyclic\ \ Gated) = \frac{E\left[(X_i^i)^2\right] - E(X_i^i)}{2\lambda_i E(X_i^i)}(1+\rho_i). \tag{65}$$

Accordingly, we can use Eq. (65) to calculate separately $E(W_i|up)$ and $E(W_i|down)$ in the Elevator-Gated scheme. $E(W_i)$ is then given by the weighted sum of $E(W_i|up)$ and $E(W_i|down)$, Eq. (31).

The number of customers found by the server at channel i at a polling instant to that channel in the up direction, $X_i^i(up)$, is the number of customers that have arrived at that channel during the time interval starting from the moment the server has last entered the channel in the down direction, until its first return in the up direction. That is,

$$X_i^i(up) = A_i\left(Y_i^{(2)} + \sum_{r=1}^{i-1}\left(Y_r^{(2)} + Y_r^{(1)}\right)\right)$$

Thus, the waiting time in the up direction is given by Eq.(65) with $X_i^i(up)$ replacing X_i^i. , In a similar manner, denoting by $X_i^i(down)$ the number of customers found by the server at channel i when polled in the down direction, we write

$$X_i^i(down) = A_i\left(Y_i^{(1)} + \sum_{r=i+1}^{N}\left(Y_r^{(1)} + Y_r^{(2)}\right)\right)$$

so that the waiting time in the down direction is given by substituting $X_i^i(down)$ in Eq. (65).

To complete the evaluation of $E(W_i)$ we turn to calculate the terms $E(X_i^i)$ and $E\left[(X_i^i)^2\right]$ for both the up and down directions.

5.4 Generating Functions

Using the same definitions as in section 4.4, but with respect to the Elevator-Gated case, we write

$$F_i^{up}(\underline{z}) \stackrel{\text{def}}{=} E\left[\prod_{j=1}^{N} z_j^{X_i^j(up)}\right], \quad i = 1, \dots, N. \tag{66}$$

The evolution of the state of the system in the up direction is described by

$$X_{i+1}^j(up) = \begin{cases} X_i^j(up) + A_j(\theta_i^{up}) + A_j\left(V_i(X_i^i(up))\right) & j \neq i \\ \\ A_i(\theta_i^{up}) + A_i\left(V_i(X_i^i(up))\right) & j = i \end{cases} \tag{67}$$

where $V_i(n)$ is the total time to serve n customers at channel i. Thus,

$$F_{i+1}^{up}(\underline{z}) = E\left[\prod_{j=1}^{N} z_j^{X_{i+1}^j(up)}\right]$$

$$= \tilde{\theta}_i^{up}\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right) F_i^{up}\left(z_1, z_2, ..., z_{i-1}, \tilde{V}_i\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right), z_{i+1}, ..., z_N\right),$$

$$i = 1, \dots, N-1. \tag{68}$$

For $X_1^i(up)$,

$$X_1^j(up) = \begin{cases} X_1^j(down) + A_j\left(V_1(X_1^1(down))\right) & j \geq 2 \\ \\ A_1\left(V_1(X_1^1(down))\right) & j = 1 \end{cases} \tag{69}$$

therefore,

$$F_1^{up}(\underline{z}) = F_1^{down}\left(\tilde{V}_1\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right), z_2, ..., z_N\right) \tag{70}$$

The PGFs of the system-state in the down direction are similarly defined as

$$F_i^{down}(\underline{z}) \stackrel{\text{def}}{=} E\left[\prod_{j=1}^{N} z_j^{X_i^j(down)}\right], \quad i = 1, 2, \dots, N. \tag{71}$$

and the evolution of the states

$$X_{i-1}^{j}(down) = \begin{cases} X_i^j(down) + A_j(\theta_{i-1}^{down}) + A_j\left(V_i(X_i^i(down))\right) & j \neq i \\ \\ A_i(\theta_{i-1}^{down}) + A_i\left(V_i(X_i^i(down))\right) & j = i \end{cases} \tag{72}$$

$$X_N^j(down) = \begin{cases} X_N^j(up) + A_N\left(V_N(X_N^N(up))\right) & j \leq N-1 \\ \\ A_N\left(V_N(X_N^N(up))\right) & j = N \end{cases} \tag{73}$$

It follows that

$$F_{i-1}^{down}(\underline{z}) = E\left[\prod_{j=1}^{N} z_j^{X_{i-1}^j(down)}\right]$$

$$= \tilde{\theta}_{i-1}^{down}\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right) F_i^{down}\left(z_1, ..., z_{i-1}, \tilde{V}_i\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right), z_{i+1}, ..., z_N\right),$$

$$i = 2, \ldots, N \tag{74}$$

$$F_N^{down}(\underline{z}) = F_N^{up}\left(z_1, z_2, ..., z_{N-1}, \tilde{V}_N\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right)\right) \tag{75}$$

Let $E(X_i^j(up)) = f_i^{up}(j) = \frac{\partial F_i^{up}(\underline{z})}{\partial z_j}\Big|_{\underline{z}=1}$ and $E(X_i^j(down)) = f_i^{down}(j) = \frac{\partial F_i^{down}(\underline{z})}{\partial z_j}\Big|_{\underline{z}=1}$.

Taking derivatives of $F_i^{up}(\underline{z})$ and $F_i^{down}(\underline{z})$, we get $2N^2$ equations in the $2N^2$ unknowns $f_i^{up}(j)$ and $f_i^{down}(j)$, $j = 1, 2, \ldots, N$, as follows

$$\begin{aligned} &f_{i+1}^{up}(j) = f_i^{up}(j) + \lambda_j\left[E(\theta_i^{up}) + f_i^{up}(i)E(V_i)\right], \quad 1 \leq i \leq N-1,\ i \neq j \\ &f_{j+1}^{up}(j) = \lambda_j\left[E(\theta_j^{up}) + f_j^{up}(j)E(V_j)\right] \\ &f_1^{up}(j) = f_1^{down}(j) + \lambda_j f_1^{down}(1)E(V_1), \quad j \neq 1 \\ &f_1^{up}(1) = \lambda_1 f_1^{down}(1)E(V_1) \\ &f_{i-1}^{down}(j) = f_i^{down}(j) + \lambda_j\left[E(\theta_{i-1}^{down}) + f_i^{down}(i)E(V_i)\right], \quad 2 \leq i \leq N,\ i \neq j \end{aligned} \tag{76}$$

$$f_{j-1}^{down}(j) = \lambda_j \left[E(\theta_{j-1}^{down}) + f_j^{down}(j) E(V_j) \right]$$
$$f_N^{down}(j) = f_N^{up}(j) + \lambda_j f_N^{up}(N) E(V_N), \quad j \neq N$$
$$f_N^{down}(N) = \lambda_N f_N^{up}(N) E(V_N) \tag{77}$$

The second moments of $X_i^j(up)$ and $X_i^j(down)$ are calculated with the aid of the second derivatives. Define $f_i^{up}(j,k) = \left.\frac{\partial^2 F_i^{up}(\underline{z})}{\partial z_j \partial z_k}\right|_{\underline{z}=1}$ and $f_i^{down}(j,k) = \left.\frac{\partial^2 F_i^{down}(\underline{z})}{\partial z_j \partial z_k}\right|_{\underline{z}=1}$, then,

$$E\left[X_i^i(up)^2\right] = f_i^{up}(i,i) + f_i^{up}(i)$$
$$E\left[X_i^i(down)^2\right] = f_i^{down}(i,i) + f_i^{down}(i) \quad .$$

$f_i^{up}(i,i)$ and $f_i^{down}(i,i)$ are obtained by solving $2N^3$ equations in the $2N^3$ variables for $f_i^{up}(j,k)$ and $f_i^{up}(j,k)$, $\quad i,j,k = 1,\ldots,N$. For $i = 1,2,\ldots,N-1$ we use Eq. (68) to get

$$\left.\frac{\partial^2 F_{i+1}^{up}(\underline{z})}{\partial z_j \partial z_k}\right|_{\underline{z}=1} = \left[\frac{\partial^2 \tilde{\theta}_i^{up}}{\partial z_j \partial z_k} F_i^{up} + \frac{\partial \tilde{\theta}_i^{up}}{\partial z_k} \times \frac{\partial F_i^{up}}{\partial z_j} + \frac{\partial \tilde{\theta}_i^{up}}{\partial z_j} \times \frac{\partial F_i^{up}}{\partial z_k} + \tilde{\theta}_i^{up} \frac{\partial^2 F_i^{up}}{\partial z_j \partial z_k} \right]_{\underline{z}=1}$$

where

$$\frac{\partial^2 \tilde{\theta}_i^{up}}{\partial z_j \partial z_k} = \begin{cases} \lambda_j \lambda_k E((\theta_i^{up})^2) & j \neq k \\ \\ \lambda_j^2 E((\theta_i^{up})^2) & j = k \end{cases}$$

$$\frac{\partial^2 F_i^{up}(\underline{z})}{\partial z_j \partial z_k} = \begin{cases} f_i^{up}(i)\lambda_j\lambda_k E(V_i^2) + \lambda_k f_i^{up}(i,j) E(V_i) \\ +\lambda_j f_i^{up}(i,k) E(V_i) + f_i^{up}(j,k) + \\ f_i^{up}(i,i)\lambda_j\lambda_k (E(V_i))^2 & j \neq k \\ \\ f_i^{up}(i)\lambda_j^2 E(V_i^2) + 2\lambda_j f_i^{up}(i,j) E(V_i) + \\ f_i^{up}(j,j) + f_i^{up}(i,i)\lambda_j^2 (E(V_i))^2 & j = k \\ \end{cases}$$

$$\frac{\partial \tilde{\theta}_i^{up}}{\partial z_k} \times \frac{\partial F_i^{up}}{\partial z_j} = \lambda_k E(\theta_i^{up}) f_i^{up}(j)$$
$$\frac{\partial \tilde{\theta}_i^{up}}{\partial z_j} \times \frac{\partial F_i^{up}}{\partial z_k} = \lambda_j E(\theta_i^{up}) f_i^{up}(k)$$

Similar equations are derived from Eq. (70). In the same manner we use Eqs. (74) and (75) to derive the corresponding set of $2N^3$ equations in the unknowns $f_i^{down}(j,k)$.

By substituting the expressions for $E(X_i^i(up))$ and $E\left[X_i^i(up)^2\right]$ in Eq.(65) one calculates $E(W_i|up)$. Using Eq. (65) again with respect to the down cycle, the corresponding expression for $E(W_i|down)$ is obtained. Finally, $E(W_i)$ is calculating via Eq. (31).

6 Elevator Globally-Quasi-Exhaustive Scheme

In this section we study the Elevator-polling, Globally-Quasi-Exhaustive service regime. We calculate expressions for the mean occupation time of the server in each channel, and obtain explicit expressions for the mean duration of a cycle. Finally, we derive expressions for calculating the mean waiting time of an arbitrary customer in the various queues, both in the up and down directions, as well as in general.

6.1 Cycle Times

Suppose that when a cycle starts, the state of the system is $\underline{L}^{(1)} = (L_1^{(1)}, \ldots, L_N^{(1)})$. Then as was described in the Introduction, under the Elevator-polling, Globally-Quasi-Exhaustive service regime the server moves along stations $1, 2, ..., N$ (up direction), staying at channel i for the duration of $L_i^{(1)}$ $M/G_i/1$-type busy periods. That is, the server makes a "global" decision at the start of a cycle as to the sojourn time it is going to spend at each channel. At the end of the up cycle the globally new state of the system is observed, say $\underline{L}^{(2)} = (L_1^{(2)}, \ldots, L_N^{(2)})$, and the server moves in the opposite (down) direction, through stations $N, N-1, ..., 1$, staying at channel i for the duration of $L_i^{(2)}$ busy periods.

This type of polling scheme is an extension of the *cyclic* polling Globally-Quasi-Exhaustive service discipline suggested by Boxma, Levy & Yechiali [1992], and discussed by Moskovitch [1992].

Considering for a moment the above mentioned *cyclic* Globally-Quasi-Exhaustive discipline, in which the server moves from channel to channel in a *cyclic* fashion. If θ_i is the switch-over time from channel i, then it is easily seen that the cycle duration is unchanged if we alter the order of service of the channels or the order of the switching times. This follows from the fact that the number of busy periods that the server stays in each channel is determined apriory at the beginning of the cycle, and is therefore independent of any change in the order of visits to the channels. In particular, if $\theta^{up} \equiv \sum_{i=1}^{N-1} \theta_i^{up} = \sum_{i=1}^{N-1} \theta_i^{down} \equiv \theta^{down}$, then the cycle duration remains unchanged if the channels are served in the order $1, 2, ..., N$ or $N, N-1, ..., 1$. Thus, the durations of the up cycle and the down cycle in the Elevator-polling, Globally-Quasi-Exhaustive model are the same, and the entire cycle duration is equal to the *sum* of two *cyclic* Globally-Quasi-Exhaustive cycles with zero switch-over time from channel 1 to channel N.

Formally, to show the equality of the up and the down cycles we write

$$E\left\{\exp\left(-\omega C_1\right)|\underline{L}^{(1)}\right\} = E\left\{\exp\left\{-\omega\left(\sum_{i=1}^{N-1}\theta_i^{up} + \sum_{i=1}^{N}\sum_{k=1}^{L_i^{(1)}} B_{ik}\right)\right\}\right\}$$

$$= \prod_{i=1}^{N-1}\tilde{\theta}_i^{up}(\omega)\prod_{i=1}^{N}\tilde{B}_i(\omega)^{L_i^{(1)}} = \tilde{\theta}^{up}(\omega)\prod_{i=1}^{N}\tilde{B}_i(\omega)^{L_i^{(1)}}.$$

Thus,

$$\tilde{C}_1(\omega) = E\left\{\exp(-\omega C_1)\right\} = \tilde{\theta}^{up}(\omega) G_{\underline{L}^{(1)}}\left(\tilde{B}_1(\omega), \tilde{B}_2(\omega), \ldots, \tilde{B}_N(\omega)\right), \quad (78)$$

where $G_{\underline{L}^{(1)}}(\underline{z}) = E\left\{\prod_{i=1}^{N} z_i^{L_i^{(1)}}\right\}$. Similarly,

$$\tilde{C}_2(\omega) = E\left\{\exp(-\omega C_2)\right\} = \tilde{\theta}^{down}(\omega) G_{\underline{L}^{(2)}}\left(\tilde{B}_1(\omega), \tilde{B}_2(\omega), \ldots, \tilde{B}_N(\omega)\right) \quad (79)$$

Now, if $\theta^{up} = \theta^{down}$ then, probabilistically, $\underline{L}^{(1)} = \underline{L}^{(2)}$ and $C_1 = C_2$ so that all cycles are (probabilisticly) the *same* and equal to the cycle duration, C_g, of an equivalent *cyclic* Globally-Quasi-Exhaustive scheme with *zero* switch-over time from channel 1 to channel N.

Hence, in order to find the cycle duration and the sojourn time of the server at channel i in the Elevator-polling scheme, we first give a brief analysis of the *cyclic* Globally-Quasi-Exhaustive model: Let $(L_1, L_2, ..., L_N)$ be the state-vector of the system at the beginning of a cycle, where L_i denotes the number of customers present at channel i, then

$$E(L_i) = \lambda_i\left[\sum_{j=1, j\neq i}^{N} E(B_j)E(L_j) + \sum_{j=1}^{N} E(\theta_j)\right], \qquad i = 1, 2, ..., N. \quad (80)$$

where θ_i is the switch-over time required when moving from channel i to the next channel. By adding $\lambda_i E(B_i)E(L_i)$ to both sides of Eq. (80) we have

$$E(L_i)\left[1+\lambda_i E(B_i)\right]=\lambda_i\left[\sum_{j=1}^{N}E(B_j)E(L_j)+\sum_{j=1}^{N}E(\theta_j)\right]\equiv\lambda_i E(C_g),$$

$$i=1,2,...,N \quad (81)$$

where C_g denotes the cycle time in the *cyclic* Globally-Quasi-Exhaustive regime. Then,

$$E(L_i)=\lambda_i(1-\rho_i)E(C_g) \quad (82)$$

Let Y_i denote the occupation time of the server at channel i calculated from the moment the server enters the channel until the end of the switch-over time to the next channel, and let R_i be the total *net* service time in channel i. Then

$$E(Y_i)=E(R_i)+E(\theta_i)=E(B_i)E(L_i)+E(\theta_i)=\rho_i E(C_g)+E(\theta_i),$$

$$i=1,2,...,N, \quad (83)$$

Clearly,

$$E(C_g)=\sum_{i=1}^{N}E(Y_i)=\sum_{i=1}^{N}\rho_i E(C_g)+\sum_{i=1}^{N}E(\theta_i) \quad (84)$$

so that, as expected,

$$E(C_g)=\frac{\sum_{i=1}^{N}E(\theta_i)}{1-\sum_{i=1}^{N}\rho_i} \quad (85)$$

Finally, setting C_g^* as the cycle time in the *cyclic* Globally-Quasi-Exhaustive regime with $\theta_N=0$, the mean of a full cycle in the *Elevator* scheme is

$$E(C)=E(C_1)+E(C_2)=2E(C_g^*)=\frac{2\sum_{j=1}^{N-1}E(\theta_j)}{1-\sum_{j=1}^{N}\rho_i}. \quad (86)$$

6.2 Mean Waiting Times

As $C_1=C_2$,

$$E(W_i)=0.5\left[E(W_i|up)+E(W_i|down)\right] \quad (87)$$

To obtain $E(W_i|up)$ and $E(W_i|down)$ we use Eq. (6) with the appropriate terms for X_i^i and T_i. Clearly,

$$T_i(up) = L_i^{(1)} + A_i\left(M_i(L_i^{(1)})\right)$$

where $M_i(L)$ is the duration of L $M/G_i/1-$ type busy periods all distributed as B_i. $X_i^i(up)$, the number of customers at station i at polling instant in the up direction, equals $L_i^{(1)}$ plus the number of arrivals to channel i ¿From the beginning of the up cycle until the entrance time of the server. That is,

$$X_i^i(up) = L_i^{(1)} + A_i\left(\sum_{j=1}^{i-1} Y_j^{(1)}\right) = L_i^{(1)} + \sum_{j=1}^{i-1}\left(A_i(\theta_j^{up}) + A_i\left(M_j(L_j^{(1)})\right)\right)$$

Hence,

$$E(T_i(up)) = \frac{E(L_i^{(1)})}{1-\rho_i} \tag{88}$$

$$E((T_i(up)^2) = E\left[(L_i^{(1)})^2\right]\left(\frac{1}{1-\rho_i}\right)^2 + E(L_i^{(1)})\left[\lambda_i^2 Var(B_i) + \frac{\rho_i}{1-\rho_i}\right] \tag{89}$$

$$E\left(X_i^i(up)T_i(up)\right)$$

$$= E\left[\left(L_i^{(1)} + \sum_{j=1}^{i-1}\left(A_i(\theta_j^{up}) + A_i\left(M_j(L_j^{(1)})\right)\right)\right)\left(L_i^{(1)} + A_i\left(M_i(L_i^{(1)})\right)\right)\right]$$

$$= E\left[\left[L_i^{(1)}\right]^2 + L_i^{(1)}\left(A_i\left(M_i(L_i^{(1)})\right)\right) + L_i^{(1)}\sum_{j=1}^{i-1} A_i(\theta_j^{up}) + L_i^{(1)}\sum_{j=1}^{i-1} A_i\left(M_j(L_j^{(1)})\right)\right.$$
$$\left. + A_i\left(M_i(L_i^{(1)})\right)\sum_{j=1}^{i-1} A_i(\theta_j^{up}) + A_i\left(M_i(L_i^{(1)})\right)\sum_{j=1}^{i-1} A_i\left(M_j(L_j^{(1)})\right)\right]$$

$$= E\left[(L_i^{(1)})^2\right] + \lambda_i E(B_i) E\left[(L_i^{(1)})^2\right] + \lambda_i E(L_i^{(1)})\sum_{j=1}^{i-1} E(\theta_j^{up}) + \lambda_i^2 E(L_i^{(1)}) E(B_i)\sum_{j=1}^{i-1} E(\theta_j^{up})$$
$$+ \lambda_i\sum_{j=1}^{i-1} E(B_j) E(L_i^{(1)} L_j^{(1)}) + \lambda_i^2 E(B_i)\sum_{j=1}^{i-1} E(B_j) E(L_i^{(1)} L_j^{(1)})$$

Thus,

$$E\left(X_i^i(up)T_i(up)\right) = \frac{E\left[\left(L_i^{(1)}\right)^2\right]}{1-\rho_i} + \lambda_i \left(\frac{1}{1-\rho_i}\right) \sum_{j=1}^{i-1} \left[E(\theta_j^{up})E(L_i^{(1)}) + E(L_i^{(1)}L_j^{(1)})E(B_j)\right] \tag{90}$$

Substituting results (88), (89) and (90) in Eq. (6), we obtain

$$E(W_i|up) = \frac{\frac{E\left[\left(L_i^{(1)}\right)^2\right]}{(1-\rho_i)^2} + E(L_i^{(1)})\left[\lambda_i^2 Var(B_i) + \frac{\rho_i}{1-\rho_i}\right] - \frac{E(L_i^{(1)})}{1-\rho_i}(1+\rho_i)}{2\lambda_i \frac{E(L_i^{(1)})}{1-\rho_i}}$$

$$+\frac{\frac{E\left[\left(L_i^{(1)}\right)^2\right]}{1-\rho_i} + \lambda_i\left(\frac{1}{1-\rho_i}\right)\sum_{j=1}^{i-1}\left[E(\theta_j^{up})E(L_i^{(1)}) + E(L_i^{(1)}L_j^{(1)})E(B_j)\right]}{2\lambda_i \frac{E(L_i^{(1)})}{1-\rho_i}}$$

$$-\frac{\frac{E\left[\left(L_i^{(1)}\right)^2\right]}{(1-\rho_i)^2} + E(L_i^{(1)})\left[\lambda_i^2 Var(B_i) + \frac{\rho_i}{1-\rho_i}\right]}{2\lambda_i \frac{E(L_i^{(1)})}{1-\rho_i}}$$

Summing the first and third terms of the above equation, we have

$$E(W_i|up) = \frac{\rho_i E\left[\left(L_i^{(1)}\right)^2\right] + E(L_i^{(1)})(1-\rho_i)\left[\rho_i(1-\rho_i)\lambda_i^2 Var(B_i) + \rho_i^2\right]}{2\lambda_i(1-\rho_i)E(L_i^{(1)})}$$

$$+\frac{E\left[\left(L_i^{(1)}\right)^2\right] + \lambda_i \sum_{j=1}^{i-1}\left[E(\theta_j^{up})E(L_i^{(1)}) + E(L_i^{(1)}L_j^{(1)})E(B_j)\right]}{2\lambda_i E(L_i^{(1)})} \tag{91}$$

$E(W_i|down)$ is derived similarly by using Eq.(6) with $T_i(down)$ and $X_i^i(down)$:

$$T_i(down) = L_i^{(2)} + A_i\left(M_i(L_i^{(2)})\right)$$

and

$$X_i^i(down) = L_i^{(2)} + A_i\left(\sum_{j=i+1}^{N} Y_j^{(2)}\right) = L_i^{(2)} + \sum_{j=i+1}^{N}\left(A_i(\theta_{j-1}^{down}) + A_i(M_j(L_j^{(2)}))\right)$$

¿From the discussion above, when $\theta^{up} = \theta^{down}$, the number of customers present at the various channels at the beginning of an up cycle or a down cycle are probabilistically the same. Thus, denoting by L_i^*, the number of customers present at channel i at the beginning of cycle C_g^*, $E(L_i^{(1)}) = E(L_i^{(2)}) = E(L_i^*)$ and $E\left[\left(L_i^{(1)}\right)^2\right] = E\left[\left(L_i^{(2)}\right)^2\right] = E\left[(L_i^*)^2\right]$. Therefore, the total waiting time is given by substituting $E(W_i|up)$ and $E(W_i|down)$ in Eq. (87), with L_i^* replacing of $L_i^{(1)}$ and $L_i^{(2)}$. Suppressing the stars from all L_i^* and C_g^*, we write

$$E(W_i) = \frac{\rho_i E(L_i^2) + E(L_i)(1-\rho_i)\left[\rho_i(1-\rho_i)\lambda_i^2 Var(B_i) + \rho_i^2\right]}{2\lambda_i(1-\rho_i)E(L_i)}$$
$$+\frac{E(L_i^2) + \lambda_i \sum_{j=1}^{i-1}\left[E(\theta_j^{up} L_i) + E(L_j L_i)E(B_j)\right]}{4\lambda_i E(L_i)}$$
$$+\frac{E(L_i^2) + \lambda_i \sum_{j=i+1}^{N}\left[E(\theta_{j-1}^{down} L_i) + E(L_j L_i)E(B_j)\right]}{4\lambda_i E(L_i)} \tag{92}$$

As

$$E_{L_i}\left\{\lambda_i E\left[\sum_{j=1, j\neq i}^{N} E(B_i)L_i L_j + \sum_{j=1}^{i-1}\theta_j^{up} L_i + \sum_{j=i+1}^{N}\theta_{j-1}^{down} L_i\right]\right\} = E(L_i^2),$$

we have,

$$E(W_i) = \frac{\rho_i E(L_i^2) + E(L_i)(1-\rho_i)\left[\rho_i(1-\rho_i)\lambda_i^2 Var(B_i) + \rho_i^2\right]}{2\lambda_i(1-\rho_i)E(L_i)} + \frac{E(L_i^2)}{2\lambda_i E(L_i)} \tag{93}$$

To complete the evaluation of $E(W_i)$ (as given by Eq.(93)), it is just left to obtain $E(L_i^2)$.

6.3 Generating Functions

Let $(L_1^{(1)}, L_2^{(1)}, \ldots, L_N^{(1)})$ be the vector state of the system at the beginning of an up cycle, and let $(L_1^{(2)}, L_2^{(2)}, \ldots, L_N^{(2)})$ be the system's state at the beginning of the following down cycle (or vica verca). Then,

$$E\left[z_1^{L_1^{(2)}}, z_2^{L_2^{(2)}}, \ldots, z_N^{L_N^{(2)}} \middle| L_1^{(1)}, L_2^{(1)}, \ldots, L_N^{(1)}\right]$$

$$= E\left[\exp\left[-\sum_{j=1}^{N}\lambda_j(1-z_j)\left(\sum_{k=1k\neq j}^{N} R_k^{(1)}+\theta\right)\right]\right]$$

$$= \tilde{\theta}\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right)\prod_{j=1}^{N}\left[\prod_{k=1k\neq j}^{N}\tilde{R}_k^{(1)}(\lambda_j(1-z_j))\right] \tag{94}$$

where $R_k^{(m)}$, the total net service time during a visit of channel k, is $R_k^{(m)} = \sum_{i=1}^{L_k} B_{ki}$, $(m=1,2)$. Therefore, $\tilde{R}_k(\omega|L_k) = \left[\tilde{B}_k(\omega)\right]^{L_k}$, and

$$E\left[z_1^{L_1^{(2)}}, z_2^{L_2^{(2)}}, \ldots, z_N^{L_N^{(2)}}\,\middle|\, L_1^{(1)}, \ldots, L_N^{(1)}\right] = \tilde{\theta}\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right)\prod_{j=1}^{N}\prod_{k=1k\neq j}^{N}\left[\tilde{B}_k(\lambda_j(1-z_j))\right]^{L_k^{(1)}}.$$

However,

$$\prod_{j=1}^{N}\prod_{k=1k\neq j}^{N}\left[\tilde{B}_k(\lambda_j(1-z_j))\right]^{L_k^{(1)}} =$$

$$= \left[\prod_{j=1j\neq 1}^{N}\tilde{B}_1(\lambda_j(1-z_j))\right]^{L_1^{(1)}}\left[\prod_{j=1j\neq 2}^{N}\tilde{B}_2(\lambda_j(1-z_j))\right]^{L_2^{(1)}}\cdots\left[\prod_{j=1j\neq N}^{N}\tilde{B}_N(\lambda_j(1-z_j))\right]^{L_N^{(1)}}.$$

Setting, as before, $\underline{L} = \underline{L}^{(1)} = \underline{L}^{(2)}$, the generating function of the system's state at the beginning of a cycle is given by

$$E\left[\prod_{j=1}^{N} z_j^{L_j}\right] = G_{\underline{L}}(\underline{z}) = \tilde{\theta}\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right)E\left[\prod_{k=1}^{N}\left[\prod_{j=1j\neq k}^{N}\tilde{B}_k(\lambda_j(1-z_j))\right]^{L_k}\right]$$

$$= \tilde{\theta}\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right)G_{\underline{L}}\left(\delta_1(\underline{z}),\ldots,\delta_N(\underline{z})\right) \tag{95}$$

where $\delta_k(\underline{z}) = \prod_{j=1,j\neq k}^{N}\tilde{B}_k(\lambda_j(1-z_j))$. Now, taking derivatives, we obtain again the set of equations (80), leading to Eq. (82): $E(L_i) = \lambda_i(1-\rho_i)E(C_g)$. That is, the total number of customers found by the server at channel i at the beginning of a cycle is equal to the number of customers who have arrived at that channel while the server was away. Setting $\gamma(\underline{z}) = \sum_{j=1}^{N}\lambda_j(1-z_j)$, and recursively define $\underline{\delta}^{(0)} = \underline{z}$, $\underline{\delta}^{(1)}(\underline{z}) = (\delta_1(\underline{z}), \delta_2(\underline{z}), \ldots, \delta_N(\underline{z}))$, $\underline{\delta}^{(m)}(\underline{z}) =$

$\underline{\delta}\left(\underline{\delta}^{(m-1)}(\underline{z})\right)$ $m = 2, 3, \ldots$, then, by iterating Eq. (95), we find, for every $M = 1, 2, \ldots$

$$G_{\underline{L}}(\underline{z}) = \prod_{m=0}^{M-1} \tilde{\theta}\left(\gamma\left(\underline{\delta}^{(m)}(\underline{z})\right)\right) G_{\underline{L}}\left(\underline{\delta}^{(M)}(\underline{z})\right) \tag{96}$$

It can be shown, similarly to the calculation in Boxma, Levy & Yechiali [1992], that
$\lim_{M\to\infty} \underline{\delta}^{(M)}(\underline{z}) = \underline{1}$ and that the infinite product $\prod_{m=0}^{\infty} E\left[e^{-\gamma\left(\underline{\delta}^{(m)}(\underline{z})\right)\theta}\right]$ converges if $\rho < 1$. Hence,

$$G_{\underline{L}}(\underline{z}) = \prod_{m=0}^{\infty} \tilde{\theta}\left(\gamma\left(\underline{\delta}^{(m)}(\underline{z})\right)\right) \tag{97}$$

The moments of the L_k's can be now calculated in the regular manner.

7 Elevator-Polling, Globally-Gated Service Regime

The cyclic-polling, Globally-Gated (GG) service regime was introduced and studied by Boxma, Levy & Yechiali [1992]. Under the *cyclic* GG procedure, at the *start* of each cycle, all customers present in the various queues are marked, and those customers are the only ones to be served during this cycle. Customers arriving in the middle of a cycle will be marked at the beginning of the *next* cycle, and will be served during that cycle. Boxma, Levy & Yechiali derived the LST of the cycle time, C, and obtained explicit formulae for $E(C)$ and $E(C^2)$. Not surprisingly, it was shown, once more, that $E(C) = \left[\sum_{i=1}^{N} E(\theta_i)\right]/(1-\rho)$, where θ_i is the switch-over time from channel i to channel $i+1$. They also derived the LST of $W_i(cyclic)$, the waiting time at queue i, and obtained an *explicit* formula for $E\left(W_i\left(cyclic\right)\right)$, namely,

$$E\left(W_i\left(cyclic\right)\right) = \left[1 + 2\sum_{j=1}^{i-1}\rho_j + \rho_i\right]\frac{E(C^2)}{2E(C)} + \sum_{j=1}^{i-1}\theta_j \tag{98}$$

Furthermore, using Eq. (98), it was shown that

$$E\left(W_1\left(cyclic\right)\right) < E\left(W_2\left(cyclic\right)\right) < \ldots < E\left(W_N\left(cyclic\right)\right).$$

Altman, Khamisy &Yechiali [1992] then studied the Elevator-polling with GG service regime. If $\theta_i^{up} = \theta_i^{down}$, then, as in the Globally-Quasi-Exhaustive case, all up and down cycles are probabilisticly the same. Utilizing this fact and result (98) they showed that, for the Elevator-GG model, mean waiting times at *all* channels are the *same*, and equal to

$$E(W) = (1+\rho)\frac{E(C^2)}{2E(C)} + \frac{E(\theta)}{2} \tag{99}$$

This is the only known nonsymetric polling scheme that achieves such a 'fairness' phenomenon.

7.1 Cycle Times and Generating functions

In this section we extend the analysis given by Altman, Khamisy & Yechiali [1992]. We will use the notation of previous sections.

The LSTs of the cycle times and the Generating Functions of $\underline{L}^{(1)}$ and $\underline{L}^{(2)}$ are related to each other as follows.

$$E\left\{e^{-\omega C_1}|\underline{L}^{(1)}\right\} = \tilde{\theta}^{up}(\omega)\prod_{j=1}^{N}\tilde{V}_j^{L_j^{(1)}}(\omega)$$

so that $\tilde{C}_1(\omega) = \tilde{\theta}^{up}(\omega)G_{\underline{L}^{(1)}}\left(\tilde{V}_1(\omega), \tilde{V}_2(\omega), \ldots, \tilde{V}_N(\omega)\right)$. Also,

$$G_{\underline{L}^{(1)}}(\underline{z}) = E_{C_2}E\left[\prod_{j=1}^{N} Z_i^{L_i^{(1)}}|C_2\right] = E_{C_2}\left[\exp\left\{-\sum_{j=1}^{N}\lambda_j(1-z_j)C_2\right\}\right] = \tilde{C}_2\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right).$$

Similarly, $G_{\underline{L}^{(2)}}(\underline{z}) = \tilde{C}_1\left(\sum_{j=1}^{N}\lambda_j(1-z_j)\right)$. Thus,

$$\tilde{C}_1(\omega) = \tilde{\theta}^{up}(\omega)\tilde{C}_2\left(\sum_{j=1}^{N}\lambda_j\left(1-\tilde{V}_j(\omega)\right)\right), \tag{100}$$

and

$$\tilde{C}_2(\omega) = \tilde{\theta}^{down}(\omega)\tilde{C}_1\left(\sum_{j=1}^{N}\lambda_j\left(1-\tilde{V}_j(\omega)\right)\right). \tag{101}$$

Hence,

$$\tilde{C}_1(\omega) = \tilde{\theta}^{up}(\omega)\tilde{\theta}^{down}\left(\delta(\omega)\right)\tilde{C}_1\left(\delta^{(2)}(\omega)\right) \tag{102}$$

where, $\delta(\omega) = \sum_{j=1}^{N} \lambda_j \left(1 - \tilde{V}_j(\omega)\right)$, and $\delta^{(0)}(\omega) = \omega$; $\delta^{(m)}(\omega) = \delta\left(\delta^{(m-1)}(\omega)\right)$, $m = 1, 2, 3, \ldots$ Iterating, and noticing that $\lim_{m\to\infty} \delta^{(m)}(\omega) = 0$, so that $\lim_{m\to\infty} \tilde{C}_1\left(\delta^{(m)}(\omega)\right) = 1$ (see Boxma, Levy & Yechiali [1992]), we finally have

$$\tilde{C}_1(\omega) = \prod_{m=0}^{\infty} \tilde{\theta}^{up}\left(\delta^{(2m)}(\omega)\right) \tilde{\theta}^{down}\left(\delta^{(2m+1)}(\omega)\right) \tag{103}$$

Similarly,

$$\tilde{C}_2(\omega) = \prod_{m=0}^{\infty} \tilde{\theta}^{down}\left(\delta^{(2m)}(\omega)\right) \tilde{\theta}^{up}\left(\delta^{(2m+1)}(\omega)\right) \tag{104}$$

Now, if $\theta^{up} = \theta^{down}$, then all up and down cycles are the same, and equal to C_1. ¿From Eq. (103) and (104),

$$E(C_1) = E(\theta^{up}) + \rho E(C_2) \quad and \quad E(C_2) = E(\theta^{down}) + \rho E(C_1).$$

Hence,

$$E(C_1) = \frac{E(\theta^{up}) + \rho E(\theta^{down})}{1-\rho^2} \quad and \quad E(C_2) = \frac{E(\theta^{down}) + \rho E(\theta^{up})}{1-\rho^2} \tag{105}$$

Clearly,

$$E(C) = E(C_1) + E(C_2) = \frac{E(\theta^{up}) + E(\theta^{down})}{1-\rho} \tag{106}$$

It readily follows now that

$$E(L_i^{(1)}) = \lambda_i E(C_2), \quad E(L_i^{(2)}) = \lambda_i E(C_1) \tag{107}$$

and

$$E(Y_i^{(1)}) = E(L_i^{(1)})E(V_i) + E(\theta_i^{up}) = \rho_i E(C_2) + E(\theta_i^{up}) \tag{108}$$

$$E(Y_i^{(2)}) = \rho_i E(C_1) + E(\theta_i^{down}) \tag{109}$$

7.2 Mean Waiting Times

We can use again Eqs. (6) and (31) to evaluate $E(W_i)$. In order to calculate $E(W_i|up)$ consider an arrival K to channel i during an up cycle. K will be served only during the *following* down cycle. Hence, the number of customers, served at queue i during the server's visit in which K is being served, is

$T_i = A_i(C_1)$, while the number of customers *present* when the server enters channel i is $X_i^i = T_i + A_i\left(\sum_{j=i+1}^{N} Y_j^{(2)}\right)$. Thus, as $E(T_i) = \lambda_i E(C_2)$ and $E(T_i^2) = \lambda_i^2 E(C_1^2) + \lambda_i E(C_1)$, the first term in (6) (when calculating $E(W_i|up)$) is given by $\frac{E(C_1^2)}{2E(C_1)}(1+\rho)$. (The probabilistic interruption of this term will become apparent in the sequel). Now, the *numerator* of the second term of (6) is given by $E\left(A_i(C_1)A_i\left(\sum_{j=i+1}^{N} Y_j^{(2)}\right)\right)$. Observe that $Y_j^{(2)}$ and C_1 are *dependent* since $Y_j^{(2)} = \sum_{j=1}^{A_j(C_1)} V_{jk} + \theta_{j-1}^{down}$.

We can complete the derivation in this manner, calculating similarly $E(W_i|down)$, but instead we choose to use a direct approach, applying arguments similar to those in Boxma, Levy & Yechiali [1992].

Consider again our customer K. His waiting time is composed of:

(i) The residual part of the up cycle, C_1^R.

(ii) The service time of all customers who arrived at queues $i+1, i+2, \ldots, N$ during the up cycle in which K arrives.

(iii) The switch over times of the server on its way down from channel N to channel i.

(iv) The service time of all customers who have arrived at channel i during the past part, C_1^P, of the up cycle in which K arrives.

Thus,

$$E(W_i|up) = E(C_1^R) + \sum_{j=i+1}^{N} \rho_j \left[E(C_1^P) + E(C_2^R)\right] + \sum_{j=i}^{N-1} E(\theta_j^{down}) + \rho_i E(C_1^P) \tag{110}$$

Since $E(C_1^P) = E(C_1^R) = \frac{E(C_1^2)}{2E(C_1)}$, we have (see also Altman, Khamisy & Yechiali [1992]),

$$E(W_i|up) = \left(1 + 2\sum_{j=i+1}^{N} \rho_j + \rho_i\right) E(C_1^R) + \sum_{j=i}^{N-1} E(\theta_j^{down}) \tag{111}$$

Similarly,

$$E(W_i|down) = \left(1 + 2\sum_{j=1}^{i-1} \rho_j + \rho_i\right) E(C_2^R) + \sum_{j=1}^{i-1} E(\theta_j^{up}) \tag{112}$$

$E(W_i)$ is readily obtained by substituting results (111), (112), (105) and (106) in Eq. (31). However, in order to complete the calculation we need to evaluate $E(C_1^2)$ and $E(C_2^2)$. By differentiating (100) and (101) we get

$$E(C_1^2) = E\left((\theta^{up})^2\right) + 2\rho E(\theta^{up})E(C_2) + \rho^2 E(C_2^2) + E(C_2)\sum_{j=1}^{N}\lambda_j E(V_j^2) \quad (113)$$

$$E(C_2^2) = E\left((\theta^{down})^2\right) + 2\rho E(\theta^{down})E(C_1) + \rho^2 E(C_1^2) + E(C_1)\sum_{j=1}^{N}\lambda_j E(V_j^2) \quad (114)$$

$E(C_1^2)$ and $E(C_2^2)$ are easily derived from (113) and (114) so that $E(C_1^R)$ and $E(C_2^R)$ are readily calculated. Now, $E(W_i)$ is completely determined.

In the case where $\theta = \theta^{up} = \theta^{down}$, then $C_1 = C_2$, so that $E(C_1) = E(C_2) = \frac{E(\theta)}{(1-\rho)}$ and

$$E(C_1^2) = E(C_2^2) = \frac{1}{1-\rho^2}\left[E(\theta^2) + \frac{2\rho[E(\theta)]^2}{(1-\rho)} + \frac{\left(\sum_{j=1}^{N}\lambda_j E(V_j^2)\right)E(\theta)}{(1-\rho)}\right] \quad (115)$$

Then, with $E(C_1^R) = E(C_2^R)$ we obtain

$$E(W_i|\theta^{up} = \theta^{down}) = (1+\rho)E(C_1^R) + \frac{E(\theta)}{2} \quad (116)$$

That is, *all* mean waiting times are equal.

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