Homework 1

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1. (First attempt at phase estimation).
   - Let U be a unitary operator. Prove all its eigenvalues have absolute value one.
   - The circuit in Figure 1 is fed with an eigenvector v of the unitary operator U. M is a measurement in the standard basis. We know that the eigenvalue associated with v is some $\lambda = e^{2\pi i\theta}$ for some $\theta \in [0, 1]$.

     What is the measurement result as a function of λ and as a function of $\cos(\theta)$?
   - Build a circuit approximating $\cos(\theta)$ with δ additive accuracy and success probability $1 - \varepsilon$. What is the complexity of the circuit as a function of ε and δ (you may assume that U has complexity 1).

2. The Bell basis has four states:

$$\beta_{0,0} = \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle] \quad , \quad \beta_{0,1} = \frac{1}{\sqrt{2}}[|01\rangle + |10\rangle],$$
$$\beta_{1,0} = \frac{1}{\sqrt{2}}[|00\rangle - |11\rangle] \quad , \quad \beta_{1,1} = \frac{1}{\sqrt{2}}[|01\rangle - |10\rangle]$$

   Show that they form an orthonormal basis.

   The circuit in Figure 2 appears as a sub-circuit of the teleportation protocol (M is again a measurement in the standard basis). Show that the circuit is a measurement in the Bell basis.

3. Prove that the measurement of a controlled bit commutes with the control operation. I.e., that the two circuits in Figure 4 are equivalent. What is the result of the teleportation protocol if we do not execute the measurements?

4. The circuit in Figure 3 runs two teleportation circuits in parallel. Alice holds the first four qubits and bob the remaining two. Initially, the third and fifth qubits are an EPR pair, and so do the fourth and sixth qubits.

   Prove that the circuit teleports a general vector in the four dimensional space.