Homework 2

Amnon Ta-Shma
Dept. of Computer Science
Tel Aviv University

1. A measurement M distinguishes ρ_1 from ρ_2 with zero error, if it has three possible answers $1, 2, quit$, and whenever it answers a non-quit value it is correct. We say M succeeds with probability p if for every input from $\{\rho_1, \rho_2\}$ it answers the correct value with probability at least p.

   - Find the bast value p with which a zero-error measurement can distinguish $|0\rangle\langle 0|$ from $|+\rangle\langle +|$.
   - find a three-dimensional measurement that achieves this value.

2. Show that a density matrix has rank 1 iff it represents a pure-state.

3. (Classical coin flipping). Alice and Bob don't trust each other and want to toss a fair coin over the telephone. Alice and Bob are probabilistic TMs. A protocol is a multi-round, private-coin interactive protocol. I.e., for any given history it specifies the player who plays next and the message that player should send (the message may depend on the player's private random coins). It also specifies when the protocol terminates, and the value of the game (i.e., who won) when that happens.

   Show that in every such protocol there is a player that can cheat and force a win (with probability 1).

4. (Quantum coin flipping I) Consider the following quantum protocol for Alice and Bob.

   - (Round 1) Alice tosses a fair coin $a \in \{0, 1\}$. If $a = 0$ Alice sends Bob $\phi_0 = |0\rangle$ while if $a = 1$ Alice sends Bob $\phi_1 = |+\rangle$.
   - (Round 2) Bob sends a random bit $b \in \{0, 1\}$.
   - (Round 3) Alice sends Bob a.
   - (The game value) Bob measures the state ϕ he received in the first round according to the basis $\left\{\phi_a, \phi_a^{\perp}\right\}$. If the result is not ϕ_a Bob wins, otherwise, the value of the game is $a \oplus b$.

     Show that:

     – If both players are honest the result is unbiased.
     – Assume that Alice is honest and Bob is not. Find the optimal strategy for Bob. What is the probability Bob wins the game under this strategy?
     – Now assume Bob is honest and Alice is not. Show that no matter what Alice's strategy is, there is always some constant probability Bob wins.