Homework 4

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1. (Continued fractions) For positive numbers $a_0, \ldots, a_k$ denote $[a_0] = a_0$, $[a_0, \ldots, a_k] = a_0 + \frac{1}{[a_1, \ldots, a_k]}$ for $k > 0$. Assume $[a_0, \ldots, a_n] = \frac{p_n}{q_n}$. The following is a recursive expression for p_n, q_n:

$$p_0 = a_0 \ , \ q_0 = 1,$$

$$p_1 = a_0 a_1 + 1 \ , \ q_1 = a_1,$$

$$p_n = a_n p_{n-1} + p_{n-2} \ , \ q_n = a_n q_{n-1} + q_{n-2}$$

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   - Write the continued fraction of $\alpha = 179/32$.
   - Prove the above recursion. (You may use the simple observation that $[a_0, \ldots, a_n] = [a_0, \ldots, a_{n-1} + 1/a_n]$).
   - Show how to find the continued fraction expansion of a rational input $\alpha = \frac{a}{b}$ in time polynomial in the input length.

2. Let

$$A \;=\; \frac{1}{2} \begin{pmatrix} 1 & \frac{7}{25} \\ \frac{7}{25} & 1 \end{pmatrix}$$

   Prove that A is positive and find $\sqrt{A}$ and $\log(A)$.

3. An automorphism of a graph G is an isomorphism from G to itself. The graph automorphism problem, GAUT, is given a graph determine wether it has a non-trivial automorphisms or not. Reduce GAUT to a HSP on $\mathbb{S}_n$.

4. Let w be a known root of unity. Show an efficient quantum circuit for the transformation $|j, k\rangle \rightarrow w^{jk}|j, k\rangle$. You may use any one, two or three qubit gates you wish.

5. Let Angle be the following promise problem:

   Input : Two pure states ϕ_1, ϕ_2, $0 \leq \alpha < \beta \leq 1$.

   Yes instances : $|\langle \phi_1 | \phi_2 \rangle| \leq \alpha$

   No instances : $|\langle \phi_1 | \phi_2 \rangle| \geq \beta$

   Design a quantum circuit that accepts Yes instances with probability at least p, and No instances with probability at most q for some $q < p$.

6. Show that $Q^{||}(EQ) = O(\log n)$.
   Hint: Use an error correcting code together with the previous question.