Problem set 12 - non-uniform computation

out: 13/1/15
due: 9/2/15

This exercise contains a few question on uniform vs. non-uniform computation for those of you who haven't seen it before.

Notation for this exercise: We say a uniform TM machine $M(x;\cdot)$ accepts a language $L$ with non-uniform advice $\{a_n\}$, if for every $x \in \{0,1\}^*$, $x \in L$ iff $M(x, a_{|x|}) = 1$. Notice that the advice depends only on the input length and not on the specific input itself.

We say $L \in DTime(t(n))|f(n)$, if there exists a uniform machine $M(x;\cdot)$ in $DTime(t(n))$ that accepts $L$ with non-uniform advice $\{a_n\}$ and $|a_n| \leq f(n)$. I.e., on inputs of length $n$, $M$ gets $f(n)$ bits of advice (that depend on the input length only) and solves $L$ in deterministic time $t(n)$. Define

$$\mathsf{P}|Poly \quad = \quad \bigcup_{k_1,k_2} \mathsf{DTIME}(n_1^k)|n^{k_2}$$

Let $Size(s(n))$ denote the set of languages $L$ that can be solved by a non-uniform family of circuits $\{C_n\}$ of size $s(n)$.

1. Prove that $\mathsf{P}|Poly = Size(poly)$.
2. Find a language $L$ that *cannot* be solved by any uniform TM, but can be solved by circuits of size 1 (i.e., in $\mathsf{DTIME}(1)|1$).
3. Prove that $\mathsf{BPP} \subseteq \mathsf{P}|Poly$.
4. Prove that if $\mathsf{NP} \subseteq \mathsf{P}|log$ then $\mathsf{NP} = P$