MATH 4022 Graph Theory (Fall '10)

Instructor: Asaf Shapira

Home Assignment 1

Due date: 09/23/10

Please submit organized and well written solutions!

Problem 1. Prove that every graph with $n > 1$ vertices and at least $2n$ edges contains a cycle of length at most $2 \log_2 n$.

Problem 2. Show that G is bipartite if and only if it has no *induced* cycle of odd length.

Problem 3. Let T be a tree and let $T_1, \ldots, T_k$ be a collection of sub-trees of T. Show that if for every $i < j$ we have $V(T_i) \cap V(T_j) \neq \emptyset$ then $\bigcap_{i=1}^{k} V(T_i) \neq \emptyset$.

Problem 4. Let $d_1, \ldots, d_n$ be a sequence of integers. Show that this sequence represents the degrees of the vertices of some tree on n vertices if and only if $\sum_{i=1}^{n} d_i = 2n - 2$ and $d_1, \ldots, d_n \geq 1$.

Problem 5. Prove that in a connected graph G every two paths of maximum length share a vertex.

Problem 6. Let T be a tree on t vertices. Show that there is an ordering $\{v_1, \ldots, v_t\}$ of the vertices of T such that for every $2 \leq i \leq t$ vertex v_i has exactly one neighbor in the set $\{v_1, \ldots, v_{i-1}\}$. Use this fact to show that if G has min-degree at least $t - 1$, then it contains every tree on t vertices as a subgraph.