

# MATH 4022 Graph Theory (Fall '10)

Instructor: Asaf Shapira

## Home Assignment 2

Due date: 10/12/10

Please submit organized and well written solutions!

**Problem 1.** Let  $G$  be a 3-regular graph. Show that its edge-connectivity equals its vertex-connectivity.

**Problem 2.** Compute the number of labeled spanning trees of the complete bipartite graph  $K_{m,n}$ .

**Problem 3.** Compute the number of labeled trees in which all degrees are either 1 or 3.

**Problem 4.** Let  $G$  be a bipartite graph on vertex sets  $A = \{a_1, \dots, a_n\}$  and  $B = \{b_1, \dots, b_m\}$ . Suppose every vertex in  $A$  has degree *at least*  $q$  and every vertex in  $B$  has degree *at most*  $r$ . Show that  $B$  contains  $n$  disjoint sets  $S_1, \dots, S_n$ , each of size at least  $\lfloor q/r \rfloor$  such that for every  $1 \leq i \leq n$  vertex  $a_i$  is connected to all the vertices in  $S_i$ .

**Problem 5.** Derive Hall's Theorem from the König-Egerváry Theorem.