MATH 4022 Graph Theory (Fall '10)

Instructor: Asaf Shapira

Home Assignment 3

Due date: 11/04/10

Please submit organized and well written solutions!

Problem 1. Let $a_1, \ldots, a_{sr+1}$ be a sequence of $s \cdot r + 1$ distinct numbers. Show that this sequence either contains an increasing sequence of $r+1$ numbers or a decreasing sequence of $s+1$ numbers.

Hint: Use Dilworth's Theorem (or its dual).

Problem 2. Derive the König-Egerváry Theorem from Dilworth's Theorem and from Menger's Theorem.

Problem 3. Let G be a bipartite graph on vertex sets A and B. Show that the size of the largest matching in G is equal to

$$|A| - \max_{S \subseteq A}(|S| - |N(S)|)$$

Problem 4. Show that every $(2k+1)$-regular $2k$-edge connected graph has a perfect matching.

Problem 5. For a path P in a directed path, let $t(P)$ be the last vertex of the path. Let G be a directed graph and let k denote the size of the largest independent set in G. Let $P_1, \ldots, P_s$ be a collection of s paths covering the vertices of G. Show that there is a collection of r paths $Q_1, \ldots, Q_r$ covering the vertices of G where $r \leq k$ and $\{t(Q_1), \ldots, t(Q_r)\} \subseteq \{t(P_1), \ldots, t(P_s)\}$.

Problem 6. Let G be a connected graph and for $e, f \in E(G)$ define $e \sim f$ if either $e = f$ or e, f belong to a common cycle of G. Prove that $\sim$ is an equivalence relation.