

MATH 4022 Graph Theory (Fall '10)

Instructor: Asaf Shapira

Home Assignment 5

Due date: 12/7/10

Please submit organized and well written solutions!

Problem 1. Show that if $\chi(G) = k$ then G has at least $\binom{k}{2}$ edges.

Problem 2. Let G be a *bipartite* graph with min-degree δ . Show that we can color its edges using δ colors so that each vertex is adjacent to at least one edge of each of the δ colors.

Problem 3. Show that an n -vertex tree has $k(k-1)^{n-1}$ legal k -colorings. Use this to show that a connected simple n -vertex graph has $k(k-1)^{n-1}$ legal k -colorings if and only if it is a tree.

Problem 4. Show that if G is k -critical then G is $(k-1)$ -edge connected.

Problem 5. Show that there is a constant $\epsilon > 0$ with the following property: if G is an n -vertex graph with at least $(1-\epsilon)\binom{n}{2}$ edges, then in any 2-coloring of its edges, one of the color classes contains a monochromatic copy of K_{10} .

Problem 6. Show that for any integer k there is an integer $n_0(k)$ such that for all $n \geq n_0(k)$ in every k -coloring of the integers $1, 2, \dots, n$, one of the color classes contains 4 integers a, b, c, d satisfying $a + b + c = d$.