Problem 1. Show that if we randomly throw $n$ balls into $n$ bins by choosing for each ball $d$ possible bins and placing the ball in the least loaded one, then the maximum load is whp at most $\ln \ln n / \ln d + O(1)$.

Problem 2. Suppose we modify Papadimitriou’s random-walk algorithm for 2-CNF, and instead of starting from an arbitrary assignment, we start the algorithm on a random assignment. What is the expected running time of the new algorithm?

Problem 3. Modify Schöning’s random-walk algorithm so that it would be able to decide (with high probability) in time poly($n$)($3/2)^n$ if a 4-CNf formula is satisfiable.

Problem 4. Consider the following balls-and-bins process; we start by throwing $n$ balls into $n$ bins. From that point on, at each iteration we remove every bin that is occupied by the balls thrown at the previous iteration, and then throw $n$ “new” balls into the remaining bins. The process ends when there are no more bins left. Show that the expected number of iterations performed by the process is $O(\log^* n)$.

**Hint:** Break the process into several “sub-steps” as in the coupon collector problem.

Problem 5. A 2-coloring of a graph is called $\Delta$-good if it does not create monochromatic triangles\(^1\). Consider the following algorithm for finding a $\Delta$-good coloring of a graph $G = (V, E)$: Starting from an arbitrary 2-coloring of $V(G)$, while there are monochromatic triangles, pick one such triangle $\Delta$, randomly pick one of the 3 vertices of $\Delta$ and change the color of that vertex.

Show that if $G$ is 3-colorable, then with high probability, this algorithm will find a $\Delta$-good coloring of $G$ after a polynomial number of recoloring iterations.

**Hint:** Start by convincing yourself that a 3-colorable graph indeed has a $\Delta$-good coloring. Also, since the problem of finding a $\Delta$-good coloring in arbitrary graphs is NP-hard, you will have to “use” the assumption that the input is 3-colorable.

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\(^1\) A triangle in $G$ is monochromatic if its three vertices are colored by the same color.