Problem 1. Give a deterministic $7/8$-approximation algorithm for MAX-3-CNF.

Problem 2. Let $S$ be a sample space of $n$ random variables which are uniformly distributed over $\mathbb{F}_4$ and 4-wise independent. Show how to use $S$ in order to construct a sample space of size $|S|$ consisting of $2n$ random variables which are uniformly distributed over $\mathbb{F}_2$ and 4-wise independent.

Hint: Recall the representation of elements of $\mathbb{F}_4$ as elements of $(\mathbb{F}_2)^2$.

Problem 3. Let $M$ be an $r \times n$ matrix over $\mathbb{F}_2$. Let $I(M)$ denote the largest integer $I$ such that every set of $I$ columns of $M$ are linearly independent over $\mathbb{F}_2$. In the following items all computations are over $\mathbb{F}_2$.

1. Suppose we pick $x$ from $(\mathbb{F}_2)^r$ uniformly at random and define a collection of random variables $\{X_1, \ldots, X_n\}$, by setting $X_i$ to be the $i^{th}$ entry of $xM$.
   Show that $\{X_1, \ldots, X_n\}$ are $k$-wise independent unbiased $0/1$ bits if and only if $I(M) \geq k$.

2. Let $C(M) = \{x \in (\mathbb{F}_2)^n : Mx = 0\}$ and $\text{wt}(x)$ denote the number of non-zero entries of $x$.
   Show that $I(M) + 1 = \min_{x \neq 0 \in C(M)} \text{wt}(x)$.

3. Show that $I(M) + 1 = \min_{x,y \in C(M)} d_H(x,y)$, where $d_H(x,y)$ denotes the Hamming distance between $x$ and $y$.

Problem 4. We have shown that if $X_1, \ldots, X_n$ are $\pm 1$ random-variables which are 4-wise independent, then $\mathbb{P}[\sum_{i=1}^n X_i \geq c\sqrt{n}] \geq c$ for some absolute $c > 0$. We will show here that one cannot replace the 4-wise independence condition with 3-wise independence.

1. Let $M'$ be the $k \times 2^k$ matrix whose columns are all possible 0/1 vectors. Let $M$ be the $(k + 1) \times 2^k$ matrix obtained by adding to $M$ a row whose entries are all 1s. Suppose we randomly pick $x \in (\mathbb{F}_2)^{k+1}$ and set the random variables $X_1, \ldots, X_{2^k}$ to be the entries of $xM$.
   Show that $X_1, \ldots, X_{2^k}$ are 3-wise independent.

   Hint: Use the first item from the previous question.

2. Let $Y_i = 1 - 2X_i$. Show that $Y_1, \ldots, Y_{2^k}$ are 3-wise independent $\pm 1$ random variables and yet $\mathbb{P}[\sum_{i=1}^{2^k} Y_i > 0] = 2^{-k}$.