Problem 1. Show that Feedback Vertex Set is at least as hard to approximate as Vertex Cover. In other words, show that if there is a polynomial time $c$-approximation algorithm for Feedback Vertex Set then there is one also for Vertex Cover.

Problem 2. Let $k$-SC-1 denote the usual Set-Cover problem restricted to instances in which every set has at most $k$ elements, and $k$-SC-2 denote the usual Set-Cover problem restricted to instances in which each element belongs to at most $k$ sets. Give a $(\ln(k) + 1)$-approximation algorithm for $k$-SC-1 and a $k$-approximation algorithm for $k$-SC-2.

Problem 3. A tournament is a directed graph obtained from a complete graph by orienting its edges.
- Show that if a tournament contains a cycle, then it contains one of length 3.
- Give a 3-approximation for the Feedback Vertex Set (FVS) problem in tournaments.
- Give an $O(3^k n^2)$ time algorithm for checking if a tournament has a FVS of size $k$.

Problem 4. Design an $O(f(k) \cdot n^2)$ time algorithm for deciding if an $n$-vertex graph has a vertex-cover of size $k$, where $f(k)$ should satisfy the recurrence $f(k) = f(k-1) + f(k-3)$.

Hint: Observe that Vertex-Cover is easy when $G$ has maximum degree 2, and when some vertex has degree 3 we can use it to get two recursion calls of sizes at most $k-1$ and $k-3$.

Problem 5. Give an $O(k! mn)$ algorithm for deciding if a directed graph with $m$ edges and $n$ vertices has a simple cycle of length $k$. The algorithm may be randomized.