CS 6550 Algorithms (Fall '10)

Instructor: Asaf Shapira

Home Assignment 3

Due date: 11/04/10

Please submit organized and well written solutions!

Problem 1. Show that Feedback Vertex Set is at least as hard to approximate as Vertex Cover. In other words, show that if there is a polynomial time c-approximation algorithm for Feedback Vertex Set then there is one also for Vertex Cover.

Problem 2. Let k-SC-1 denote the usual Set-Cover problem restricted to instances in which every set has at most k elements, and k-SC-2 denote the usual Set-Cover problem restricted to instances in which each element belongs to at most k sets. Give a $(\ln(k) + 1)$-approximation algorithm for k-SC-1 and a k-approximation algorithm for k-SC-2.

Problem 3. A tournament is a directed graph obtained from a complete graph by orienting its edges.

- Show that if a tournament contains a cycle, then it contains one of length 3.
- Give a 3-approximation for the Feedback Vertex Set (FVS) problem in tournaments.
- Give an $O(3^k n^2)$ time algorithm for checking if a tournament has a FVS of size k.

Problem 4. Design an $O(f(k) \cdot n^2)$ time algorithm for deciding if an n-vertex graph has a vertex-cover of size k, where $f(k)$ should satisfy the recurrence $f(k) = f(k-1) + f(k-3)$.

Hint: Observe that Vertex-Cover is easy when G has maximum degree 2, and when some vertex has degree 3 we can use it to get two recursion calls of sizes at most $k-1$ and $k-3$.

Problem 5. Give an $O(k!mn)$ algorithm for deciding if a directed graph with m edges and n vertices has a simple cycle of length k. The algorithm may be randomized.