MATH 7016 Combinatorics (Spring '09)

Instructor: Asaf Shapira

Home Assignment 2

Due date: 02/12/09

Please submit organized and well written solutions!

Problem 1. Let $W(k)$ be the smallest integer n such that in every 2-coloring of the first n integers one of the color classes contains a k-term arithmetic progression. Show that $W(k) \geq 2^{k/2}$.

Problem 2. Prove that for every r and d, there is an integer $N = N(r,d)$ such that for every $n \geq N$, every r-coloring of the first n integers contains a replete affine d-cube (an affine d-cube is replete if all its elements are distinct). Do not use van-der Waerdan's Theorem.

Problem 3. Let $A_d(n)$ be the size of the largest subset of $\{1, \ldots, n\}$ containing no affine d-cube.

- Show that if a set of integers X contains no 3-term arithmetic progression and no replete affine d-cube, then X contains no affine d-cube.
- Use the previous item to prove that for every $\epsilon > 0$ and every large enough $n \geq n_0(\epsilon)$ we have
$$A_d(n) \geq n^{(1-\epsilon)(1-d/2^{d-1})}$$

Problem 4. For an integer $k \geq 3$, let $R_k(n)$ denote the size of the largest subset of $\{1, \ldots, n\}$ which contains no k-term arithmetic progression. Let $C_k(n)$ denote the smallest number of colors that suffices for coloring $\{1, \ldots, n\}$ such that no color class contains a k-term arithmetic progression. Show that

$$\frac{n}{R_k(n)} \leq C_k(n) \leq \frac{30n \log n}{R_k(k)} .$$