MATH 7018 - Probabilistic Combinatorics (Fall '09)

Instructor: Asaf Shapira

Home Assignment 1

Due date: 9/10/09

Please submit organized and well written solutions!

Problem 1. Suppose $n > 4$ and let H be an n-uniform hypergraph with at most $4^{n-1}/3^n$ edges. Prove that there is a coloring of the vertices of H by 4 colors so that in every edge all 4 colors are represented.

Problem 2. Prove that there is an absolute constant $c > 0$ with the following property; Let A be an $n \times n$ matrix with pairwise distinct entries. Then there is a permutation of the rows of A so that no column in the permuted matrix contains an increasing sub-sequence of length at least $c\sqrt{n}$.

Problem 3. Let F be a finite collection of binary strings of finite lengths and assume no member of F is a prefix of another one. Let n_i denote the number of strings of length i in F. Prove that

$$\sum_i \frac{n_i}{2^i} \leq 1 \, .$$

Problem 4. Let $G = (V, E)$ be a bipartite graph on n vertices with a list $S(v)$ of more than $\log_2 n$ colors associated with each vertex $v \in V$. Prove that there is a proper coloring of G assigning to each vertex v a color from its list $S(v)$.

Problem 5. Prove that every set A of n nonzero integers contains two *disjoint* subsets $B_1, B_2 \subseteq A$, so that $|B_1| + |B_2| > 2n/3$ and each set B_i is sum-free (that is, there are no $b_1, b_2, b_3 \in B_i$ so that $b_1 + b_2 = b_3$.)