Problem 1. Prove that the number of surjective (i.e. onto) mappings from \([n]\) to \([k]\) is given by
\[
\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n
\]. Use this to deduce that:

- \[\sum_{i=0}^{n}(-1)^i\binom{n}{i}(n-i)^n = n!\].
- \[\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n = 0\] when \(k > n\).
- \[S(n, k) = \frac{1}{k!} \sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n\], where \(S(n, k)\) are the Stirling numbers of the second kind.

Problem 2. Consider the number of ways of coloring the integers \(\{1, \ldots, 2n\}\) using the colors red/blue in such a way that if \(i\) is colored red then so is \(i - 1\). Deduce the identity
\[
\sum_{k=0}^{n}(-1)^k\binom{2n-k}{k}2^{2n-2k} = 2n + 1
\]

Problem 3. Show that the number of subsets of size \(k\) of \(\{1, \ldots, n\}\) which contain no pair of consecutive integers is given by \(\binom{n-k+1}{k}\).

Problem 4. Let \(A_1, \ldots, A_n\) be a family of \(n\) sets. Show that
\[
|\bigcup_{i=1}^{n} A_i| \geq \sum_{1 \leq i \leq n} |A_i| - \sum_{1 \leq i < j \leq n} |A_i \cap A_j|
\]
and
\[
|\bigcup_{i=1}^{n} A_i| \leq \sum_{1 \leq i \leq n} |A_i| - \sum_{1 \leq i < j \leq n} |A_i \cap A_j| + \sum_{1 \leq i < j < k \leq n} |A_i \cap A_j \cap A_k|
\]