Topics in Extremal Combinatorics (0366.4996)- Fall '21

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Home Assignment 1

Due date: 2/11/21

Please submit organized and well written solutions!

Problem 1. Show that for every $0 < \alpha, \beta, \gamma < 1$ and $\delta > 0$ there is ϵ so that if K is a 3-partite graph on vertex sets A, B, C and $d(A, B) = \alpha$, $d(B, C) = \beta$, $d(A, C) = \gamma$ and (A, C) and (B, C) are ϵ regular, then K contains $|A||B||C|(\alpha\beta\gamma \pm \delta)$ copies of K_3.

Problem 2. Suppose H is an n-vertex graph of maximum degree Δ. We proved in class that if G has $4n$ vertices and density at least $1 - \frac{1}{8\Delta}$ then G contains a copy of H. We also proved that if G has $C(\delta, \Delta) \cdot n$ vertices and G is bi-$(c(\delta, \Delta), \delta)$-dense then G has a copy of H (for an appropriate $c(\delta, \Delta)$). Do the proofs of these two results work also when H is only assumed to be Δ-degenerate?

Problem 3. Show that $r(5, n) \leq Cn^4/\log^3 n$ and that there is an n-vertex K_4-free graph G satisfying $\alpha(G) \leq cn^{2/5} \log n$.

Problem 4. Prove the following "embedding lemmas" in which G is a bipartite graph with vertex sets X, Y (**Hint:** Just do it!).

- Suppose every collection of r vertices in X have at least b common neighbors in Y. Then G contains a copy of every bipartite graph with vertex sets A, B satisfying $|A| < |X|$, $|B| \leq b$ and where every vertex in B has at most r vertices in A.

- Suppose every collection of r vertices in X have at least n common neighbors in Y, and every collection of r vertices in Y have at least n common neighbors in X. Then G contains a copy of every n-vertex r-degenerate bipartite graph.