Problem 1. $G$ is an $(n,d,\lambda)$-graph if $G$ is an $n$-vertex $d$-regular graph satisfying the following property: if $S \subseteq V(G)$ is of size $\alpha n$ then
\[
|e(S) - \frac{1}{2} d\alpha^2 n| \leq \frac{1}{2} \lambda \alpha (1 - \alpha) n.
\]
Show that there are absolute constants $\beta, \delta > 0$ and $d_0$ so that the following holds for all $d \geq d_0$ and $n \geq n_0(d)$: if $G$ is an $(n,d,\delta d)$-graph then every 2-coloring of $G$ contains a monochromatic path of length $\beta n$.

Problem 2. We’ve seen that if every $U \subset V(G)$, $|U| \leq u$ satisfies $|N(U)| \geq 2|U|$, then $G$ contains $P_{3u-1}$. Prove that under the same assumption we can actually find a cycle of length at least $3u$.

Problem 3. Show that for every $\epsilon$ there is $C = C(\epsilon)$ so that if $G$ has $\epsilon n^2$ edges and no independent set of size $n/2^{\sqrt{\log n}}$ then $G$ contains a $K_4$.

Problem 4. Show that if $H$ is an $r$-degenerate bipartite graph then $ex(n,H) \leq cn^2 - \frac{1}{r}$. 

Problem 5. Show that if $G$ is a bipartite graph with $m$ edges and no isolated vertices then $r(G) \leq 2^{O(\sqrt{m})}$. 