Topics in Extremal Combinatorics (0366.4996)- Fall '21

Instructor: Asaf Shapira

Home Assignment 3

Due date: 21/12/21

Please submit organized and well written solutions!

Problem 1. G is an (n, d, λ)-graph if G is an n-vertex d-regular graph satisfying the following property: if $S \subseteq V(G)$ is of size αn then

$$\left| e(S) - \frac{1}{2} d\alpha^2 n \right| \leq \frac{1}{2}\lambda\alpha(1-\alpha)n \ .$$

Show that there are absolute constants $\beta, \delta > 0$ and d_0 so that the following holds for all $d \geq d_0$ and $n \geq n_0(d)$: if G is an $(n, d, \delta d)$-graph then every 2-coloring of G contains a monochromatic path of length βn.

Problem 2. We've seen that if every $U \subset V(G)$, $|U| \leq u$ satisfies $|N(U)| \geq 2|U|$, then G contains P_{3u-1}. Prove that under the same assumption we can actually find a cycle of length at least $3u$.

Problem 3. Show that for every ϵ there is $C = C(\epsilon)$ so that if G has ϵn^2 edges and no independent set of size $n/2^{C\sqrt{\log n}}$ then G contains a K_4.

Problem 4. Show that if H is an r-degenerate bipartite graph then $ex(n, H) \leq cn^{2-\frac{1}{8r}}$.

Problem 5. Show that if G is a bipartite graph with m edges and no isolated vertices then $r(G) \leq 2^{O(\sqrt{m})}$.