1. For a graph $G$ we write $G^2$ for the graph on the same vertex set as $G$ so that vertices $u, v$ form an edge if and only if the graph distance in $G$ between $u$ and $v$ is at most 2. Show that if $G$ has uniformly bounded degrees, then $G$ is recurrent if and only if $G^2$ is recurrent.

2. Construct an example of a local limit $(U, \rho)$ of finite planar graphs such that $U$ is almost surely recurrent, but $U^2$ is almost surely transient.

3. Let $G(n, p)$ be the random graph on $n$ vertices drawn such that each of the $\binom{n}{2}$ possible edges appears with probability $p$ independently of all other edges. Let $c > 0$ be a constant, show that $G(n, c/n)$ converges locally to a branching process with progeny distribution Poisson($c$).

4. Fix an integer $k \geq 1$. Construct an example of a sequence of finite simple planar maps $G_n$ such that $G_n$ converge locally to $(U, \rho)$ with the property that $\mathbb{E}[^{\text{deg}^k}(\rho)] < \infty$ and $U$ is almost surely transient.

5. Suppose that $G_n$ is a sequence of finite trees converging locally to $(U, \rho)$. Show that $U$ is almost surely recurrent. (Note that the degrees may be unbounded)