Planar maps, random walks and circle packing — exercise sheet 3

This assignment is due on Monday, Januray 8th, 2018.

1. For a graph G we write G^2 for the graph on the same vertex set as G so that vertices u, v form an edge if and only if the graph distance in G between u and v is at most 2. Show that if G has uniformly bounded degrees, then G is recurrent if and only if G^2 is recurrent.

2. Construct an example of a local limit (U, ρ) of finite planar graphs such that U is almost surely recurrent, but U^2 is almost surely transient.

3. Let $G(n, p)$ be the random graph on n vertices drawn such that each of the $\binom{n}{2}$ possible edges appears with probability p independently of all other edges. Let $c > 0$ be a constant, show that $G(n, c/n)$ converges locally to a branching process with progeny distribution Poisson(c).

4. Fix an integer $k \geq 1$. Construct an example of a sequence of finite simple planar maps G_n such that G_n converge locally to (U, ρ) with the property that $\mathbb{E}[\deg^k(\rho)] < \infty$ and U is almost surely transient.

5. Suppose that G_n is a sequence of finite trees converging locally to (U, ρ). Show that U is almost surely recurrent. (Note that the degrees may be *unbounded*)