Probability in 2D problem set 1
Due date: November 28, 2016

1. Independently orient each vertical edge of $\mathbb{Z}^2$ upwards with probability $\frac{1}{2}$ and downwards with probability $\frac{1}{2}$, and each horizontal edge of $\mathbb{Z}^2$ left with probability $\frac{1}{2}$ and right with probability $\frac{1}{2}$. Show that with probability 1 there is no simple infinite oriented path starting from the origin.

2. Consider bond-percolation on $\mathbb{Z}^2$ with $p = \frac{1}{2}$. Show that there exists a constant $c > 0$ such that for any integer $n \geq 1$ the probability that the origin is connected to $\mathbb{Z}^2 \setminus [-n, n]^2$ is at least $\frac{1}{2\sqrt{n}}$.

3. Show that there exist constants $C < \infty$ and $c > 0$ such that for all n the probability of the event from the last question is at most Cn^{-c}.

4. Consider bond-percolation on $\mathbb{Z}^2$ with $p = \frac{1}{2}$ and let $\alpha(n)$ be a positive integer sequence such that $\alpha(n)/n \to 0$. Show that the probability of a horizontal open crossing of n by $n + \alpha(n)$ rectangle tends to $\frac{1}{2}$.