1. Let $G$ be a transitive infinite connected graph and let $\rho$ be an arbitrary vertex of $G$. For an integer $r > 0$ denote by $\partial B_\rho(r)$ the set of vertices which have graph distance $r$ to $\rho$. Consider bond percolation on $G$ with $p_c = p_c(G)$. Show that

$$\mathbb{E}\left|\{v \in \partial B_\rho(r) : \rho \leftrightarrow v\}\right| \geq 1.$$  

2. Consider bond percolation in the box $\{-n, \ldots, n\}^3 \subset \mathbb{Z}^3$ with $p = p_c(\mathbb{Z}^3)$. Show that there exists constants $c_1, c_2$ such that for all $n$

$$P\left((0,0,0) \leftrightarrow (n,n,n)\right) \geq c_1 e^{-c_2 \log^2(n)}.$$  

3. Consider bond percolation in the slab $\{1, \ldots, n\} \times \{1, \ldots, n\} \times \{1, \ldots, n/3\} \subset \mathbb{Z}^3$ with $p = p_c(\mathbb{Z}^3)$. Denote by $p_n$ the probability that there is an open path crossing the slab in the third coordinate. Show that there exists a constant $c > 0$ such that $p_n \geq c$ for all $n$.

4. Let $\text{Maj}_1 : \{0,1\}^3 \rightarrow \{0,1\}$ be the majority boolean function on 3 bits. Recursively define for $n \geq 2$ the function $\text{Maj}_n : \{0,1\}^{3^n} \rightarrow \{0,1\}$ by setting

$$y_1 = (x_1, \ldots, x_{3^n-1})$$
$$y_2 = (x_{3^n-1+1}, \ldots, x_{2.3^n-1})$$
$$y_3 = (x_{2.3^n-1+1}, \ldots, x_{3^n})$$

and putting

$$\text{Maj}_n(x_1, \ldots, x_{3^n}) = \text{Maj}_1(\text{Maj}_{n-1}(y_1), \text{Maj}_{n-1}(y_2), \text{Maj}_{n-1}(y_3)).$$

Is $\text{Maj}_n$ asymptotically noise-sensitive? Prove your claim.

5. A boolean function $f : \{0,1\}^n \rightarrow \{0,1\}$ is called transitive if for any $i, j \in [n]$ there exists a permutation $\sigma \in S_n$ of $[n]$ such that $\sigma(i) = j$ and such that for any $x \in \{0,1\}^n$ we have that $f(x_1, \ldots, x_n) = f(x_{\sigma(1)}, \ldots, x_{\sigma(n)})$. Show that there exists some constant $c > 0$ (independent of $n$) such that any randomized adaptive algorithm $A$ determining a balanced (i.e., $\mathbb{E}f = 1/2$) transitive boolean function $f$ has revealment $\delta_A \geq cn^{-1/2}$.