COMPLEXITY: Exercise No. 2
due in two weeks

1. a. Prove that if $t(n)$ is a complexity function such that $t(n_0) \leq n_0 + 1$ for some n_0 then any language in $\mathbf{TIME}(t(n))$ is regular (a language is regular if it can be accepted by a finite automaton).
   b. Show a language L in $\mathbf{TIME}(n+2)$ which is not regular.

2. (Test 98)Is the following problem in **NL**?
   Given an undirected graph G, vertices x, y from G, and a positive integer K, does the shortest path from x to y is of length (exactly) K?

3. Prove that for any $f(n)$ and $\epsilon > 0$, $\mathbf{SPACE}(f(n)) \subseteq \mathbf{SPACE}(\epsilon f(n))$.

4. Show that a language decided by a k-tape NDTM operating within time $f(n)$ can be decided by a 2-tape NDTM in time $f(n)$ (Assume that $f(n)$ is a proper complexity function, and that $f(n) = \omega(n)$).

5. a. For a language L define $L^* = \{x_1 x_2 \cdots x_k : k \geq 0, x_1, \ldots, x_k \in L\}$. Show that if $L \in \mathbf{NP}$ then $L^* \in \mathbf{NP}$.
   b. Show that if $L \in \mathbf{P}$ then $L^* \in \mathbf{P}$.

6. True, false or open? explain: $\mathbf{SPACE}(n^3) \subseteq \mathbf{NSPACE}(n)$.

7. a. For a language L and a function $f : N \to N$ satisfying $f(n) \geq n$, define $\mathrm{Pad}(L, f(n)) = \{x\#^{f(|x|)-|x|} : x \in L\}$ where $\#$ is a symbol not in the alphabet of L. A complexity class $\mathbf{C}$ has the *padding property* if for any integer $k \geq 1$, $L \in \mathbf{C} \iff \mathrm{Pad}(L, n^k) \in \mathbf{C}$. Which of the following classes have the padding property: $\mathbf{P}, \mathbf{NP}, \mathbf{SPACE}(n)$?
   b. Prove that $\mathbf{NP} \neq \mathbf{SPACE}(n)$ and $\mathbf{P} \neq \mathbf{SPACE}(n)$.